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Creativity in Engineering Mathematical Models Through Programming

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Creativity in Engineering Mathematical Models Through Programming

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Abstract We discuss classroom activity comprising of small groups of students collaboratively tinkering with programs of dynamically manipulable figural models posing problems regarding their mathematical properties and behaviors. We analyzed data from students' discourse taken from two classroom interventions employing a framework of creative mathematical action-in-context in order to study student-generated ideas. We approached students' actions taking a fallible mathematics epistemological approach and employed constructionist and social creativity theory in our analysis. Our results show that student agency in the disciplined field of mathematical thinking need not curtail the potential for undisciplined creative action, on the contrary given appropriate tools and discursive environments it may in fact create space for actions with creative potential for students. Out of their own accord, the students in the study used generalized number theory to resolve engineering a parallelogram which can never be a rectangle and used recursion to program a model embedding geometrical progression to create a spiral based on the golden ratio.

Keywords creative mathematical actions, programming, mathematics, constructionism, social creativity

Introduction

This paper investigates student-generated creative actions in a classroom context where the teacher and the use of a particular digital medium for mathematical expression where meant to inject a fallibilist style of mathematical activity (Davis & Hersch, 1980) in an otherwise formalist mathematics type of schooling. In our theoretical frame section we firstly explain why we give central importance to mathematical epistemology in order to study the creative actions of learners. We then discuss the study of creativity within a framework recently suggested by Riling (2020) focusing on creative mathematical actions-in-context, actions with creative potential emerging within the community of a school classroom. We thus explain the kinds of activity emerging in our classroom contexts which are based on the use of a particular programmable digital medium for expressing mathematical reasoning through the modeling of animated figures. To discuss creative actions in a social discursive classroom setting we then elaborate on what we mean by a socio-technical environment, a community jointly working with a digital medium. Recognizing that, from a research point of view it is much more complex, we nevertheless felt it necessary to jointly employ these diverse theoretical constructs if we really wanted to gain insight into creativity in a classroom community as action-in-context.

So, in designing our classroom pedagogical intervention to enrich opportunity for creative actions, we adopted Riling's approach (2020) who suggests that it is hard to look for creativity in formalist mathematical contexts since creativity emerges from agency, the capacity to act independently. The kind of disciplined agency inherent in formalist mathematics restricts rather than enriches the potential for creative actions assigning importance to reason rather than insight. Riling (2020) goes as far as to suggest that even in the case of problem posing, creativity has been mainly studied in the context of given tasks to pose problems, timed activities, responses to teacher-originated challenges where the teacher, the discipline and the institution are primary authority systems. In this research we wanted to identify instances of learner creative action in a transformative educational paradigm where mathematics is seen as created by humans and characterized by combining and selecting ideas and concepts not usually connected with the problem at hand and by making decisions on which problems to pose and attempt to resolve.

Rather than addressing a particular level of creativity, such as for instance the inherent in all of us everyday 'little-C' (as proposed by Kaufmann and Beghetto, 2009 and Craft et al, 2013), we attempted to identify and describe creative actions in their transaction with a particular disciplined mathematical context pedagogically engineered to allow for undisciplined agency. We wanted to contribute to addressing the problem of how to look for and how to encourage undisciplined agency within the disciplined context of mathematical reasoning. To do that, we took a fallibilist view of mathematical activity and focused on the interactions between students, their teacher and the digital tools used as expressive media for engineering mathematical models. We wanted to consider what kind of mathematical context would allow and encourage undisciplined mathematical activity.

Framing the study

In the past 30 years or so attempts to understand, discuss and cultivate creativity in mathematics education have, albeit sometimes implicitly, taken a diversity of approaches with respect to what it means to know and do mathematics, to the nature of mathematical creativity and to the focus of attention alternating amongst person, process and product (Liljedahl & Sriraman, 2006).

Much of this work began and continues to look for students with the potential for the kind of high-impact domain-disrupting creativity expected from mathematicians' thus connecting creativity to ability and giftedness with respect to the education sector. More recently however, researchers have in parallel been looking to identify, characterize and cultivate creativity as inherent in all learners of mathematics, including youngsters. Original work by Craft (2001) suggested that beyond high-impact 'big-c' creativity it is worthwhile addressing 'little - c', a type of creativity in all of us manifested in everyday kind of situations and that it is thus valuable to look at that type of creativity in domain teaching in school. Kaufmann and Beghetto (2009), built on this idea for mathematics education and suggested that it is useful to go even further and think of mini-c, a type of creativity inherent in the generation of mathematical meaning. Pitta et al (this issue) suggest that from a pedagogical point of view it is worth finding ways to cultivate mini-c in mathematics classrooms on the way to infusing mathematical thinking in everyday little-c. However, the research has hitherto largely assumed mathematical activity as taking place in formalist mathematical schooling and has studied timed student responses to given tasks and problem solving and posing activities in context of discipline, teacher and institutional authority (Boaler, 2003, Riling, 2020).

Furthermore, the focus so far has overbearingly been on person, process or product, predominantly on the creativity of an individual even when individuals have been involved in collaborative or interactive classroom situations. In effect, researchers have looked to identify in learners the kind of creativity-as-talent originally associated with mathematicians, for instance the four-level preparation, incubation, illumination, verification process (Milos, 2016). There has been very little work focusing on creativity in connection with mathematical epistemology and thus creativity as mathematical action embedded in learner transaction in a community such as for instance within a classroom context. This research attempts to provide some insight into social student - led creativity in an informal mathematical classroom context. We thus framed the research by connecting diverse but in our view necessary perspectives of mathematical epistemology, creativity as action-in-context, the affordances of a specific context based on mathematical activity through programming and on social creativity in this particular type of context.

Creativity and fallible mathematics

In the mathematics education community mathematical creativity has mainly been addressed in two ways, by means of the 'genius' approach connecting creativity to giftedness (Mann, 2006, Sriraman, 2005, Leikin, 2013) and by means of the problem solving-posing approach (Silver, 1997) extending the search for creativity in both closed and open mathematical problems. Although never explicitly discussed so far, we would suggest that, from a fallibilist perspective at least, both approaches veer towards a Platonist view of mathematical activity (Sriraman, 2004) assuming that mathematics is out there to be discovered by the mathematician and by association, by the learner.

According to Riling, 'the fallibilist perspective has two main distinctions from formalism that have implications for the role of creativity in mathematics education. First, fallibilism positions mathematics as a totally human invention, which means that it provides a coherent rationale for creativity being a part of mathematics education. Second, fallibilist creativity does not rely on the existence of an unknowable and uncontrollable mathematical intuition that some students may lack, which means that the fallibilist perspective does not position any students as incapable of mathematical creativity' (Riling, 2020, p.14).

In the setting we studied, students were encouraged to pose problems as they emerged during discursive activity characterized by engineering artifacts in the form of programmable dynamic figural models, tinkering with the models' properties and behaviors, sharing them and discussing over them while doing so. The pedagogical motivation for generating such contexts was to free up and cultivate the kind of creativity which grows with mathematical meaning-making taking some distance from taken-as-established mathematics and traditional curriculum structures.

To address such situations, we perceived doing mathematics to densely involve a learner exposing meanings and articulations of logical thought and justifications to criticism and refutation attempts rather than exposing statements of positivist truths. We argue that epistemology matters in the study of mathematical creativity and that fallibilism (Davis & Hersch, 1980) embodies an epistemology which is pedagogically promising for the cultivation of creativity emerging from learner agency, i.e. from the students' own undisciplined decisions while engaged in a disciplined structured activity in a mathematical classroom context (Grootenboer & Jorgensen , 2009).

In order to identify creative mathematical actions in this particular context involving the programmable digital medium, we needed to adopt the well-recognized meaning-making approach to mathematical learning and a constructionist approach to doing mathematics (Noss and Hoyles, 1996). Papert (1972) provocatively argued that mathematical meaning-making is natural to learners. It is traditional schooling that somehow imposes an artificial picture of mathematics to be the practice of trying to understand the abstract products of mathematical activity rather than the activity itself. These products are in fact irrelevant to the meaning-making process of students (Kynigos, 2015). The imposition of this kind of mathematical learning denies students the opportunity and encouragement to engage in the 'logic which gives birth to concepts' (Lakatos, 1976).

Lakatos (1976) was equally provoking in his book ‘proofs and refutations’ where he analyzed mathematicians’ activity as a process of conjecture, as a public expression of thought and subsequent engagement with a cycle of refutation, redrafting and new proofs. Sriraman (2004) discusses Davis and Hersch's (1980) distinction of three approaches to mathematical activity, Platonism, Logicism and Formalism. He draws attention to a fourth school of thought, constructivism, (Ernest, 1991 in Sriraman, 2004), in the search for a 'mode of thinking or inquiry which leads to meaningful questions and to the methodology of tackling such questions' (p.21). He stresses that such mode of thinking is highly determined by culture i.e., within social interaction.

Creativity as action in context

Our aim in this study was to contribute to an understanding of creativity as meaning-making in a socio-constructionist context characterized by an engineering kind of mathematical activity, i.e. where collaborating groups co-construct and tinker with dynamic digital artifacts by means of programming and discuss their mathematical properties and behaviors in the process (Papert, 1980, Kynigos 2015).

The fallibilist approach to mathematics perceives that, rather than discovered, mathematics is created by humans. Riling suggests that in this context, rather than focusing on the level of creativity, it is meaningful to focus on human activity, to think of a creative mathematical action and to position such action in a community. She suggested that it is useful to look for mathematical action with creative potential in interaction with a context within a community (Riling, 2020, Craft et al 2013).

Riling defines a creative mathematical action as 'one that transitions a given mathematical context into a new version of mathematics by creating ways of doing or thinking about mathematics that were previously not possible for a particular community of mathematicians', Riling, 2020, p.17

In education, actions with creative mathematical potential are mostly those which learners engage in on their own accord, their own agency (Wagner, 2007). Within such a fame, for example, creative actions are those of combining concepts and ideas to resolve a problem which are usually not considered as connected or to select an unusual concept or argument to resolve a problem. Riling points out that creativity in a person can be thought of the frequency of taking creative actions but points out the importance of circumstance. Skovsmose in fact talks about learner intentions grounded in a disposition to take agency (Skovsmose, 2001). Acting on one's own accord has been juxtaposed to authority by researchers such as Anderson and Noren (2011) and Boaler (2003), the latter elaborating on Pickering's notion of 'dance of agency' in order to discuss discourse and interactions between learners and teachers within institutions as authority systems including mathematics as a domain. Agency in formalist mathematical activity is highly disciplined and researchers have maintained that it may well inhibit the emergence of creative ideas and solutions to mathematical problems.

Agency is not only the capacity to act independently but also an intention grounded in disposition as Anderson and Noren (2011) point out in their analysis of Skovsmose's (2005) take on agency. They point out that Skovsmose perceives of learners as acting and reflecting subjects in a mathematical classroom and that certain forms of communication may enhance space for agency. Anderson and Noren synthesize Skovsmose's elaboration of agency in mathematical learners with Biesta & Tedder, 2006, who refer to learners as actors in transaction with context.

In our study we explicitly looked for student agency, i.e. learners' dynamic competence to act independently and make action choices as Anderson & Noren would put it (2011). We adopted Anderson's & Noren's view of agency that is not just individual, but exercised within social practices and perceived of agency as tightly connected to context noting their quoting of Holland, Lachicotte, Skinner, & Cain (2003) “Agency lies in the improvisations that people create in response to particular situations” (p. 279). We thus needed to adopt a social-relational approach (Glăvenau, 2015) affording creativity a dynamic emergent essence within cultural settings in the context of social interaction (Craft and Jeffrey, 2008, Riling, 2020). But first a few words about the affordances of the particular context where creativity was sought in this study.

Engineering through programming mathematical models

Learning through tinkering models is based on Constructionist theory (Papert & Harel, 1991), a special kind of fallibilist mathematical activity which argues that learning occurs naturally when students take agency while making and sharing tangible artefacts (Gauntlett et at., 2009). Constructionism comprises a strong educational design element (Kynigos, 2015), where powerful mathematical ideas are embedded by pedagogical designers in special kinds of artifacts accompanied by construction units and tools, even a construction language in some cases of digital media. Such construction kits are thus designed to provide dense opportunity for students to concretize and express their ideas by designing themselves, building and engineering (Healy and Kynigos, 2010). Thus, in digital constructionist learning environments learners' agency is encouraged since they become designers using technology to build and modify artifacts which become public entities when shared with peers, while teachers act as facilitators of the process (Blikstein, 2013). This constructionist view highlights the importance of social participation in the learning process, as well as the emergent productions with usually low social impact range (Papert, 1980).

This leads us to revisit the notion of the ‘creative product’. Girvan (2014) mobilizes Vernon's (1989) definition of creativity to suggest that an artifact is necessarily created and then shared with others as 'a person’s capacity to produce new or original ideas, insights, restructurings, inventions, or artistic objects, which are accepted by experts as being of scientific, aesthetic, social, or technological value' (p.94). A creative product is often described as creative in terms of novelty and appropriateness (Amabile, 1983), however, novelty can be in terms of both historical novelty, in that it has never been thought of by anyone ever before, or psychological novelty, an idea which is novel to the individual (Boden, 2004) or to the immediate social environment. However, it can be argued that novelty is not creativity, without appropriateness to the problem to be solved. These concepts of novelty and appropriateness can also be applied to the evaluation of the ideas phase in the creative process model.

Social Creativity

So, how could we account for creative mathematical action in a classroom social setting based on fallible constructionist mathematics with a programmable dynamic modeling medium? Fischer’s (2004) approach of ‘Social Creativity’ is a theoretical framework for understanding creativity in collectives as an effect of the interaction among the members of a group, between them and the media they use (mostly digital) and the artefacts that they create. In this approach, the emergence of creativity springs as a shared expression of a group of people engaged in the joint enterprise of constructing or modifying a digital artifact or tool, rather than as a sum of individual creative actions. According to Fischer et al. (2005) the nature of creativity, in a broader sense, has four components; originality, expression, social evaluation and social appreciation. While originality is a common component in other approaches of creativity, the other three components centrally take into account the communication and the interaction within the group something which was crucial to understand in our study. Glăvenau (2015) argues for the value of social/relational theories of creativity appreciating the importance of interdependent but diverse perspectives of process and product. He stresses the temporal aspect and the dynamic view of creative potential 'creative ideation and achievement are continuous with each other, what people think about and do at time x opens up a new sphere of potential achievements at time x+1' (p. 115). Social Creativity as a theoretical framework has not been widely used in educational settings, but it is related to our broader research questions for a clearer view of students' group discourse while they were engaged in their model tinkering activities. The role of participants, as it is described by Fischer (2002) in situations where Social Creativity emerges was another reason to think of it as a legitimate framework for implementing our intervention; our students did not just use the digital media, but they contributed by modifying and redesigning them in a way that had a negotiated personal meaning for them.

So, in this study, we found it imperative to try to integrate these otherwise diverse frame in order to ask and throw some light on the following questions: what kind of disciplined mathematical classroom context may allow for undisciplined agency to become legitimate? what kind of context affordances generate dense opportunity for discursive creative mathematical action?

3 The design of the Research

This research used data from two diverse real classroom situations in one of the experimental schools in Greece. These are 62 schools with some special features such as the right for teachers to make some minor modifications to the ways the curriculum is taught or to suggest some extra activities for the students after agreement by the school board which is headed by a university pedagogy professor.

The digital medium which was used for mathematical modeling was a well-established, freely available on-line programming tool recognized by the Ministry of Education. In Greece, the on-line mathematics curriculum books are replete with links to 'micro-experiments' a significant number of which set the students the task to fix a buggy program so that it generates a generalized geometrical object such as a parallelogram or an isosceles triangle containing the respective properties by means of variable values and inter-variable functional relationships (Kynigos, 2020). Out of the 1600 - odd micro-experiments residing in the on-line mathematics curriculum books from year 3 to 11, 220 were created with this tool, called 'MaLT2'. MaLT2 embeds a Logo programming language to create 3D figural models affording dynamic manipulation of variable procedure values (http://etl.ppp.uoa.gr/malt2/). This last feature creates a sense of dynamic 'behavior' of a figural model when it is constructed by a parametric procedure. Mathematical formalism in the form of a programming language, figural representations and dynamic manipulation of generalized values are interlinked representations.

The first class we collected data from was a normal classroom of Grade-7. 26 students who got involved in an investigation with MaLT2 during their lessons. They had had experience with this already since their normal teacher had integrated some on-line MaLT2 tasks in the mathematics course. In our research school, typically, students worked in pairs trying to de-bug a given artefact and then used the bug-free model resulting from their work as an object with which to create something of their own. The students had the opportunity to communicate with their collaborating peer, and also discuss an idea at whole classroom level. This discussion took place when students wanted to make a suggestion on the task, or when the teacher though that it would be good to bring an idea of a pair of students to a whole class discussion.

Our second classroom case was within the Mathematics Club of the school. Students opted to join the club out of their own interest but perhaps also influenced by the value their parents attributed to Mathematics. In this school, students started being members of the Club in Grade-8 and could stay as members to the end of high-school (5 years). Club activities included solving problems and riddles and involved investigations in groups. 15 students from different years attended the club that year. The tasks and challenges that were addressed to them were chosen so that all of the students could get involved into investigation regardless of their Grade. In this case we focused on three Grade-11 students that were club members since their Grade-8. They were high achievers in Mathematics. As researchers, in both cases we wanted to keep an open mind on the type of meanings were related to mathematical concepts and logical thinking processes (Warr and O'Neil, 2005).

Although these situations took place in a real school setting, (researcher and researcher-teacher) had to intervene by designing and putting to action a specific learning environment, since tinkering digital artifacts by means of programming is not a common practice in schools. We saw our intervention as a cycle of design experiments informed by the results and experience from our previous studies, in terms of the Design-Based Research (DBR) framework (Bakker, 2018).

For the first class case, based on experience from previous cycles of our research on meaning-making (Diamantidis et al., 2019; Papadopoulos et al., 2016) we adopted our pedagogical technique of starting the students off by giving them what we call half-baked digital models and asking them to identify and fix the bugs we intentionally placed within (Kynigos, 2007). This was a permutation of the buggy procedures given in a micro-experiment in the curriculum book in that we were the ones choosing what bug to insert. We use the term half-baked models to describe digital artifacts which we intentionally design to be incomplete or faulty and we call this a kind of didactical engineering. At the beginning of an activity we typically challenge students to explore the reason for their buggy behavior. So, students are consciously engaged in trying to identify bugs, fix them and subsequently create their own model configurations using the fixed model as a construction unit. This latter part of the activity is loosely if at all structured by the teacher allowing for students to pose problems and make choices of what model structures to build with their fixed models. The idea is to design situations where half-baked models operate as a spark for creative mathematical-engineering ideas to emerge. In our previous research cases, the lens of analysis was mostly focused on the meaning-making process and on the discourse among students and teachers who participated in these studies. Here we addressed creativity in such a context. For the second class, the mathematics club we employed a different design for the activity by giving the students a working model and asking them to use the same programming technique to create a model of their own.

Besides the didactical engineering of group bug-fixing and model generation activity, we paid attention to the ecology in which the research took take place. What kind of classroom setting would be suitable to facilitate a creative process? Is there a specific learning ecology in which creativity could emerge spontaneously? So, apart from adopting the constructionist idea that learners act as designers, we employed a 'Social Creativity' perspective as a methodological basis for our study in order to come to grips with the ecology of our intervention.

4 Method

We analyzed the student-student and student-teacher interactions during the process of bug-fixing and model modification. In both cases the data we analyzed were produced during the research and consisted of voice recorded conversations among students and students and teachers, their actions on the computer screen that were captured with the use of a screen video recording application, all the digital artifacts that students produced and the researcher's field notes. The teacher that participated in the study was a members of the research team and kept notes from the field.

We analyzed the data following a grounded approach. We analyzed students dialogs and put their expressions in categories looking especially for instances where they demonstrated agency, making their own decision on how to proceed. So we encountered initiatives and suggestions on the selection of mathematical context to be used, on different views around the correct answer to the task, on unusual combining of mathematical concepts with regards to school mathematics, on making trials of new ideas in a construction. In both classroom cases, there were other ideas and incidents from other groups of the classroom and the mathematics club respectively that had these characteristics. However, the two episodes that we chose to elaborate on in this paper were the most indicative; students' agendas in both cases were significantly different from the initial task given to them and led to a product.

We addressed this learning environment as a meta-design situation, i.e. where participants study the ways in which given artifacts have been designed by others and engage in re-designing them to fix bugs or make changes in their behaviors. We were looking for creative actions in the process and the outcomes of this kind of environment assuming that the original half-baked artifacts would play the role of an organizer of the activity, a sparker for actions with creative potential (Fischer & Giaccardi, 2006). We took meta-design to mean that the participants should consider modification of given artifacts to be within normal classroom activity and not part of some laboratory situation.

We analyzed student discourse, dynamic manipulation and programming not only with respect to the bug-fixing process during the phase of solving 'the task' but also what they came up with doing with their programs after fixing them. We were equally interested in the 'outcome phase', and the 'process phase', studying meta-design and the discourse around it in an educational context, as an interaction that underpins creative thinking. We approached the students acting as designers and engineers, taking both as characteristics of creative engagement to tasks (Howard et al., 2008) through the posing and refining of questions before, during and after they address them.

What kind of creative mathematical actions can be identified in engineering through collaborative programming discursive situations? What kind of mathematical concepts may emerge and how would it be valuable to think of their impact range? What role do these kind of digital representations play in the emergence of creativity in such discursive environments? how can this kind of creativity cultivate mathematical meaning-making?

5 Results

Constructing a Parallelogram which can never be a rectangle.

We observed eight Grade-7 successive lessons of a regular class comprising of 26 students working in pairs each sharing a computer. In this section we focus on the communication and interaction between the students of a group, that took place during the third lesson. The task they were set was taken from the digital version of their curriculum book and came in the form of a link to the following specific MaLT2 'micro-experiment' (Kynigos, 2020). Pressing the link opened up a live MaLT2 file comprising the code of a defined procedure and an executed instance of the procedure with six consecutive values of 80 130 80 130 150 30 (see fig. 1). The procedure code was:

```
to parallelogram :a :b :c :d :f :w
  fd :a rt :f
  fd :b rt :w
  fd :c rt :f
  fd :d rt :w
end
```

The code commands an avatar (in the form of a bird) to alternate between displacement (fd) and turn (rt), each displacement leaving a linear trace. The value of each displacement and turn is however expressed in the form of a variable (e.g. :a = variable 'a'). If the task is to create a parallelogram procedure, the given procedure is buggy since a) there is no functional relationship between displacements when they should be alternately equal b) turns are alternately equal but there is no relationship between consecutive turns which should be complementary (see fig. 2). However, the values given to the students in its execution create an *instance* of a parallelogram since they are instances of the required relations. The task text asks whether the procedure always creates a parallelogram and if not, can they fix it to do so?

The intent of the exercise is for the students to manipulate the variable values in the sliders shown in fig. 1, realize that changing values results in animations breaking down the parallelogram figure and thus identify that the original figure was only an instance of what the given procedure can generate. The students can then look back into the code to search for ways to express functional relations between alternate segments and subsequent turns so they can create a procedure for a generalized parallelogram. The code in that case would be a formal description of the properties of the figure necessary for the bird to construct it, hence the engineering perspective of the properties and their implementation.

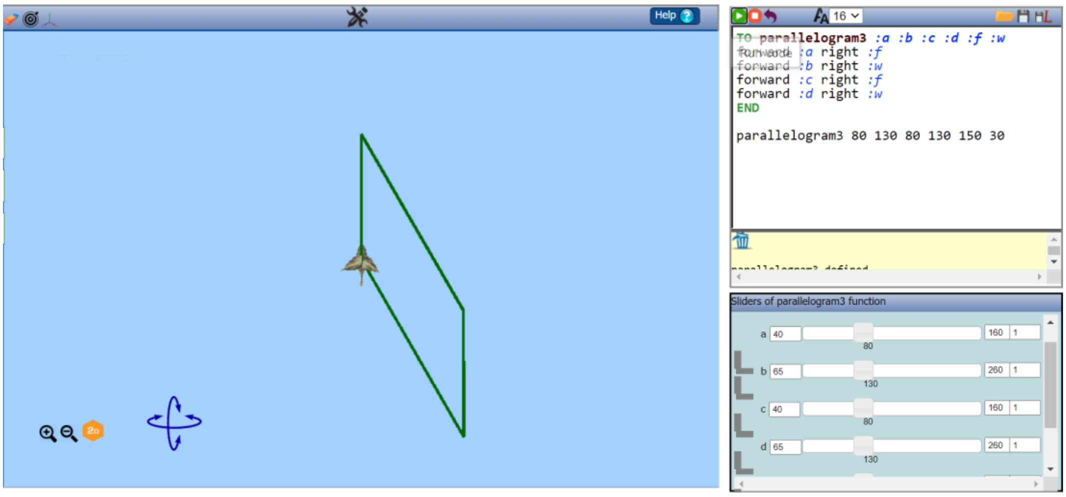


Figure 1: A parallelogram instance of a buggy procedure

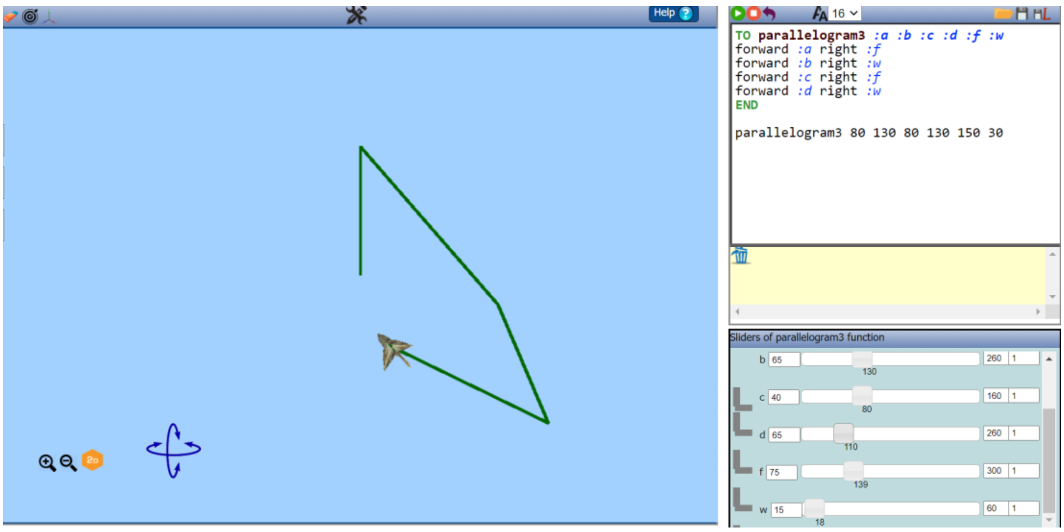


Figure 2: A random instance of the buggy procedure

The question posed to the students by text attached to the medium was ‘Can you fix the program to always make a parallelogram?’ We pick up the group's activity after two sessions when they had already found the 'solution' by looking into their textbooks and also talking to their peers. Their solution was to apply the same variable for alternative displacements (:a for first and third, :b for second and fourth), and one variable for the first turn (:f) followed by a linear relationship of 180-:f for the next and so on (fig.3). So far so good, we could characterize the activity as taking place in a mathematically disciplined engineering environment and the kinds of creative actions coming from the students as characterized by disciplined agency, solving a set task.

However, these students had second thoughts about having solved the task. They started discussing what exactly was the aim of the task, with regards to the question posed. We have enriched the original dialogs with some comments in italics, to articulate the situation.

1 S1: Ok, this is a closed one *[figure]*, but it is not always a parallelogram.

2 S2: I have seen that J and M *[they were two peers from another group]* have made a similar shape. It seems to be ok!

3 S1: But if I move this slider to 90 *[the slider of variable :f]* this is a rectangle *[fig. 3]*. the task says 'always create a parallelogram'. Is it ok if it's a rectangle or not? Is it what the questions asks from us to do?

4 S2: I don’ t know, should we ask J and M? But, wait, what is wrong if the shape was a rectangle, for a set of values?

5 S1: We ‘d better ask teacher.

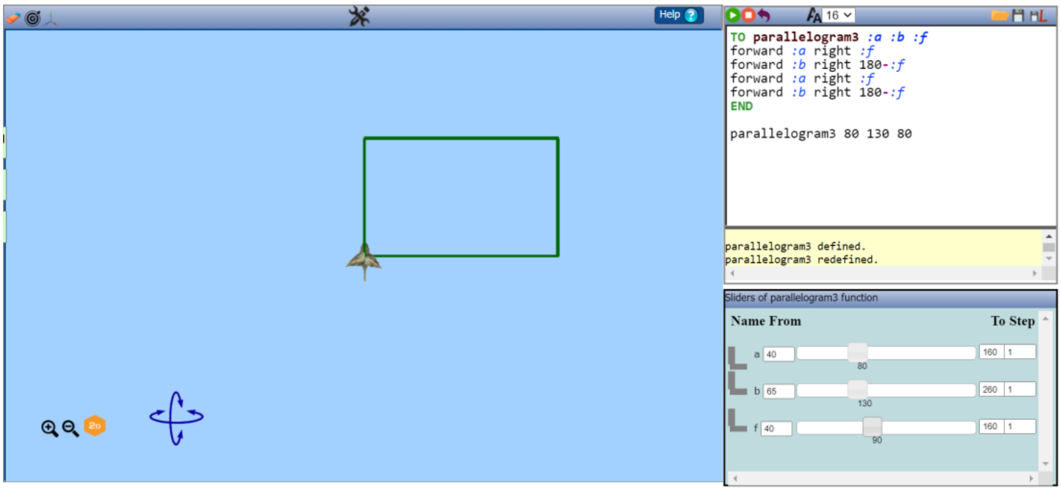


Figure 3: The program/solution that students reached to. S1 manipulated dynamically the shape arguing that it could be a rectangle.

It is always surprising to the educator how resistant the students were to accept that it is ok for a parallelogram to be a rectangle, one does not cancel out the other, in fact the latter necessarily has the propertied of the former. In their curriculum book however, these two figures appear in distinct sections with distinct naming and properties. So, even though the generalized expression they had just built obviously created either figure they actually felt that they needed to literally meet the task - to create the former only and not the latter. Since no mention was explicitly made about a rectangle then the procedure ought to *not* create such a figure. Even though the students seemed to understand class inclusion in this case they did not seem to accept it since two distinct names had been given to sub-class and class. The dilemma was expressed situated in S1's words (line 3). The next extract shows how the students became more articulate about perceiving this as a problem.

6 S1: We have a problem here. We have made this parallelogram program, however, it makes a rectangle for :f equals 90.

7 T1: Is it a problem? Why?

8 S1: Because it asks for a parallelogram, not a rectangle!

9 S2: Yes, this is a special case! Rectangle is a special case.

10 T1: So, I see you do not need a special case. Can you just ignore it?

11 S2: I do not know. If that were the case, it would have asked of a rectangle, as well.

While in line 9, it seems that S2 did think of a rectangle as a special case of parallelogram she wasn't satisfied with the inclusion as shown in line 11; the inclusion property seemed to blur when it came into use. The task did not specifically ask for 'a parallelogram or a rectangle', therefore the students assumed that the latter was not wanted. So, what we could say is that we had a situation that provoked meaning generation around parallelograms, their classes, and their properties. Now, the teacher's reaction is key here. In line 10, he seemed to join in, legitimize the students' posing of a problem. In a formalist approach he would have just told them about class inclusion and that they did not need to bother any further. Here though, he engaged in their quest as if he was part of the group.

So, the students actually set themselves the task to create a generalized parallelogram procedure excluding the case of rectangle. At first they fixed the slider range to end up in 89, so that dragging the turn slider would not get to 90. Their teacher refuted that as a solution by simply changing the range of the slider, showing that it was not a generalized solution. In the dialog which follows we can see a progressive articulation of the student-posed problem. The teachers' refutation provoked a more sophisticated articulation of the problem by the students who had got excited by the ownership - the agency - of this new task they had set themselves. They accepted that manipulating the slide-range was not legitimate, the solution has to be found in changes to the code so that a rectangle would be impossible whatever the value of the variables.

20 S1: Ok, lets see the variable of turning right, what we called it?

21 S2: It is :f.

22 S1: Oh yes, :f! The variable we already limited between 0 and 89. What can we do to avoid being equal to 90? Is there any way to set a limit with a command into the program? *[S1asks the teacher]*. Something like :f<90.

23 T1: No, you cannot do this. You should think of another way. What you can do is to use variable :f in a different way in the program.

24 S2: Which means?

25 S1: Perhaps we could use a condition, say if :f=90, then to turn right :f-1, e.g., 89.

26 S2: Or to turn right :f+1, e.g., 91..

The engineering aspect of the activity led students to try to forge links between a geometrical property and the idea of a generalized number which however could not take one specific value. This could be characterized as a selecting a mathematical idea which would be unusual to associate with parallelogram geometrical properties. Conditionals were not on the menu since the students had not been show how to use them in the programming language. When they came back to the next section, after apparently having spent time thinking about this and discussing it, S1 and S2 presented a new version of their program in which they used the commands 'right 2*:f+1' and 'right 180-(2*:f+1)' for the successive turnings. So, their solution to a generalized number excluding the value of 90 was a generalized expression of odd numbers, thus barring all even numbers including 90. They themselves explained, that turning was of odd degrees, which prevented the program from producing a rectangle under any kind of dynamic manipulation, while still producing a lot of parallelograms. So, we don't want the value of 90? exclude all even numbers and we're there! It is obvious that the understanding of class inclusion for these students was not deep enough to be sufficient to solve the set task. In a formalist mathematical environment this would have been explained and the students would have seen it easily. However, what was important here for the teacher and what was facilitated by the digital tool and the classroom norm, was the fallibilist approach to mathematical activity. It did not matter so much what triggered this problem posed by the students. What mattered was that they made a decision to pursue it, that it was not set by the teacher or the task and that what resulted was a mathematical process of propositions and refutations and a creative selection of an unusual idea to resolve it. In a formalist environment the students would not have had the space to engage in such an activity and if they did it would most likely have been unnoticed by teacher and researchers in this case.

The case of the Quadrant Spiral

In this case, we focused on a group of three Grade-11 students (S3, S4 and S5) working on MaLT2, during their after-class Mathematics' school club. They were experienced users of MaLT2, as they had started making programs on it three years prior to the study. Since then, they had been involved in activities with MaLT2, at least twice a year during the Mathematics' club session.

The sessions of the club took place in the school lab, where each group of students was working on a computer. The tasks were designed by the teacher (T2) who was responsible for running the club, adopting the role of a facilitator of students' activity during the sessions. The sparker for the incident that we are describing here was a program that used recursion. This program (fig. 4) was given to all the students, by the teacher, along with the task to make a shape of their own preference using it (fig. 4). When executed, the code creates a rectangular spiral since in each recurrence the displacement is 5 units less than the previous one. Execution with a value of 130 results in a spiral of 24 segments. The students of the focus group began to investigate the structure of recursion. The task directly gave agency over to them since they were challenged to create something of their choice using recursion. The discipline here however was to first figure out how recursion works.

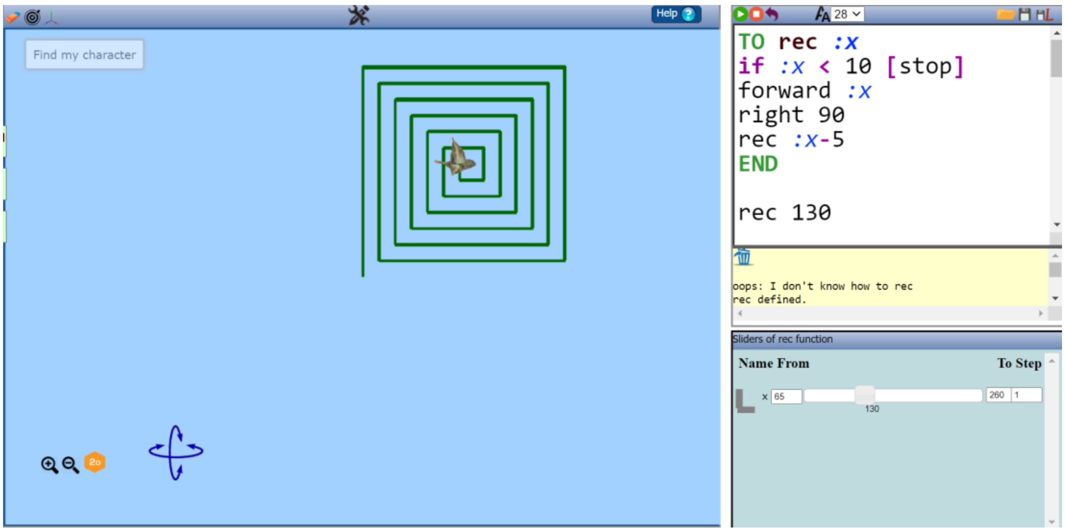


Figure 4: The program using recursion which was given as a sparker to the students.

1 S3: Is it possible that a program calls itself? I am talking about 'rec' which uses 'rec :x-5'.

2 T2: explains how the program runs in a formalist way, but stops there

3 S4: Got it! It is like repetition, but not exactly like repetition.

4 S3: Why not?

5 S5: Because there is not a fixed time of repetitions.

6 S3: But it is! Since you run the program ‘rec 130’ you know that it is going to be repeated 24 times.

7 S4: It is more than that, since every time it is repeated, the value of :x is not the same. So, it is not a real repetition.

In the extract above, students argued on the way the recursion occurs, using examples that referred to the program given by T2. They tried to establish a common way of understanding recursion, thus a shared meaning of what it really is. The key element that they had tracked down around recursion was that in every nested execution of rec, the argument of rec is not the same. This was crucial to move to the next step.

8 S4: This was the problem we had in the past when we tried to make the spiral, remember?

9 S3: Yes, we had to repeat the same pattern of commands many times in the program, just because of that problem we faced around repetition.

10 S5: Which one?

11 S3: We wanted the radius of the arc to be different, in each repetition! We could not construct a program like this with a usual repeat command.

12 S5: Maybe we can try to use this type of repetition here, which is not a real repetition, off course!

13 S4: Yes, but it will save us! We can make the program without having to use a fixed number of repetitions, which was very restrictive!

14 S5: You talk about the number of repetitions, uh?

15 S4: Yes, about the spiral’s length.

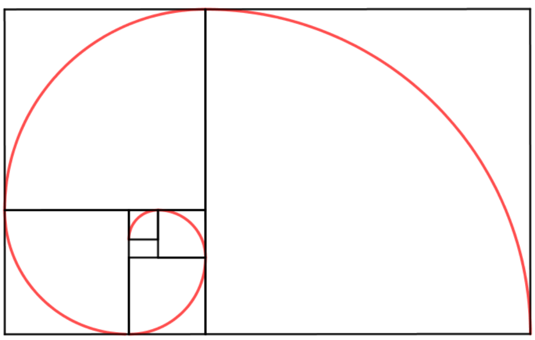
In this extract, students recalled a problem they themselves had posed one month ago (lines 8 and 9) but had failed to solve; they were trying to make a spiral made of quarter circles, like an image they had seen in a poster (fig. 5). In that image, the spiral’s quarter circles were drawn inside squares. The side of each square represented the radius of each quarter circle. As mathematicians we know that the spiral was related to a geometric series of the ‘golden ratio’ known as ‘Phi’. So, if the radius of the first quarter circle was equal to :r, then the next one should have radius equal to :r*(1+√5)/2. Students, probably driven from the image, wrote a program with a non-intrinsic approach of the arc’s construction, relying on the radius; they used ‘repeat 9 [forward (2*pi*:r)/36 right 10]’. Up next, they wanted to repeat the same pattern of commands, but with radius equal to :r*(1+√5)/2, and so on. This was a barrier to their efforts since they could not use the repeat command to make the spiral, as mentioned by S3 in line 11. To overcome this barrier, they simply iterated the commands with a different input each time respectively. This is probably what S4 mentions in line 13. The restriction that S3 said was around the length of the spiral, since it was related with how many times the pattern was repeated in the program.

16 S5: There is a small problem here. We want :r to change in every repetition. Should we set a limit on the value of :r to stop the program?

17 S4: I do not think so.. We do not want :r to be of limited set of values.

18 S3: We should think of the condition to stop the program. Does it have to be related to :r?

In line 18, student S3 posed a question, that made their conception of recursion clearer. Although in the program that T2 showed, the value of :x was connected not only to the ‘if’ condition, but to the properties of the shape (i.e., its length) as well, students did not need this relation between the conditional and the properties of the shape (as it seemed in line 16). So, the question of S3 was about using a certain aspect of a recursion program, to control the numbers of quarter circles, separate from the length of the spiral. They made it by using another variable :n (fig. 5). So, through their discourse around the mechanism of recursion, students adopted an element of it to solve a problem with personal value for them. Then they put it in use under their own agenda.



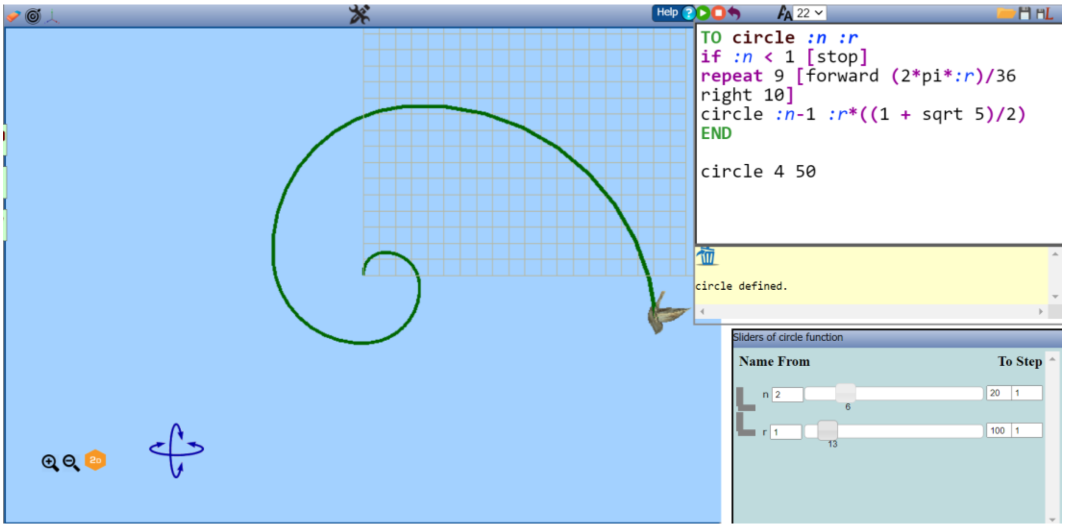


Figure 5: The image in the upper side was the sparker for students' idea to make a spiral. In the lower side, there is the spiral they made using recursion.

In this case, we believe that the students' creative action was to combine recursion with geometrical progression via the golden ratio problem. In fact they seemed to use the golden ratio as a means both to clarify how recursion works but also as a means to engineer a model based on the golden ratio using recursion to make a previously cumbersome solution much more mathematical in its expression. The creative mathematical action here was combining two distinct ideas to create a model which they themselves decided they wanted to create. The process was similar to the previous example to express a generalized value excluding 90. The only difference was the level of mathematical understanding and perhaps the impact range since connecting recursion to geometical progression can be thought of an interesting idea beyond the specific context of that particular class.

Discussion

The one thing that these two cases do not have in common is the level of mathematical concepts the students respectively chose to work on. The idea they generated and worked on in the first case was to seek for a way to express a generalized number barring one specific value in order to construct a class of objects barring a specific sub-class. Indeed, this was sparked by a superficial understanding of class inclusion which could have been generated by the curriculum structure which identified class and sub-class by name in distinct sections each with its own set of definitions, exercises etc. Even though the students said that rectangle was a specific case of a parallelogram, they assumed that the absence of reference to the former in the task question meant that the required result had to omit it.

What made the students out of their own accord wish to dig into this task further after having reached an acceptable solution? What was it about the classroom norms that gave them the space for agency and allowed for the students to *select* a mathematical idea from number theory, in other respects disconnected to the problem of properties of a geometrical figure? We suggest that Skovsmose's idea of a pedagogy enhancing space for agency is relevant here. For a researcher or a teacher, identifying creative mathematical actions would involve looking for situations where students felt it was legitimate to pose their own problems and to jointly engage in a quest to progressively articulate them and to search for a solution. In our view this is characteristic of undisciplined agency within a disciplined mathematical activity and this may provide a more focused lens with which to approach the first question of this study, what kinds of context allow for creative mathematical actions.

The problem in itself, actually makes little mathematical sense, maybe one could draw out the idea of a discontinuous function but this was not on the students' mind. This was obvious in the first reaction of their teacher in line 7. The process however, subsequently encouraged by the teacher, was mathematical in the fallibilist Lakatosian sense, the search for a generality was mathematical and the generalized expression for odd numbers in a formal language which could be inserted as code in a program to engineer a generalized figure were mathematical. The teacher's refutation of the 'solution' of simply ending the slider range at the value of 89 was also done in this spirit, to validate the students' agency in this mathematical process but at the same time encourage them to pursue the search for a generalized solution to their problem. In that sense, the teacher steered the students back to disciplined thought but in the context of their own problem. In turn, the students seemed to take on the challenge posed by this refutation, to leave the dynamic face-value representation and search for generality in terms of the formal language of the procedure. Furthermore, our point is that this was a case of possibility thinking (Craft et al, 2013) where a creative disposition at the time could well be seen as potentially generating subsequent creativity at a later time to use Glăvenau's temporal idea of alternative perspectives. If the students felt good about their investigation and solution they would be disposed to take decisions to engage in mathematical activity again. To put it in other words the impact range of constructing an odd-angled parallelogram was within this group of students, or maybe at best communicated to the class of 26 but the impact on the students' disposition may well have been much deeper.

We see the second case as an example of creative actions *combining* otherwise disconnected mathematical ideas. The students were introduced to recursion by been given a procedure generating a right-angled spiral and challenged to create a recursive procedure of their own. Their progressive perception of the essence of recursion was that it allowed mathematically expressed change at each iteration of a command. It was their idea to connect this to a prior task of effectively engineering a spiral based on the golden ratio, which they had attempted before but could only manage it with a cumbersome code writing down each iteration. Taking that perspective, their idea and solution is an interesting implementation of recursion to express geometrical progression and one could argue that if the problem was generalized it could have an impact range for mathematics beyond school mathematics. Looking at the students' activity however yields a kind of fallible mathematical activity similar to that of the first group of students. Here too the process was mathematical, the students saw a diverse implementation of an already generalized idea of a right-angled spiral and put this conjecture to use by creating a program for the quadrant arc spiral. The flow of communication in the process of engineering a digital model allowed for a mathematical problem to be expressed in this way, i.e. that recursion is a technique to express geometrical progression. Recursion was transformed from an object to analyze and understand to a tool with which certain genre of model can be built.

So, in both cases we observed evidence of students' possibility thinking in terms of posing new inquiries and asking 'what if', and we could say that disposition for creative mathematical action was apparent, in an interplay with meaning making around mathematics. Amongst the affordances of the context at hand we would highlight the kind of discourse encouraged by the teacher, a kind of pedagogical engineering of agency giving students space and legitimacy for coming up with and owning their problems. At the same time, the digital medium afforded this dance of agency, the linked representations, the sense that the students could engineer any model they liked but also needed to work out properties of models embodying generalized functional relations in order to do so. An important affordance as we see it here is that with such tools, the stakes are much lower, i.e. the consequences of creating something buggy and unexpected are insignificant since tinkering with a model was the normal way to work.

The medium, the discourse and the legitimization of creative disposition were intertwined and the teacher was only an integral part in this socio-technical culture as Fischer would agree (2004).

However, the setting of a group of students using a digital medium like MaLT2, in a learning ecology like that of the two cases, sparked discourse around making things, designing, and putting ideas in use, and led to creative mathematical actions. An affordance also worth taking into account was the progressively more articulate and precise mathematical language used by the students and encouraged by their teacher. In both cases, we can see increasing accuracy as the discussion unfolds. These expressions became more and more concrete as the discourse became more dense.

In conclusion this study left us with the conviction that it is well worth studying mathematical creativity in terms of the disposition and the density of student generated creative mathematical actions in transaction with the social and representational affordances of the context at hand. In our view this opens up many tough questions to pursue in order to understand and to encourage creative mathematical action in the classroom. How much investment is needed in time and energy and at what cost in relation to routine understanding and use of mathematical processes? Where is the individual in all this and how can individual creativity be drawn out of social engagement? And in the end, what kind of mathematical understandings can creative mathematical actions generate?

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