INTERNAL RECORD AS A CATALYST: FOURTH GRADERS' PROBLEM-SOLVING PRACTICES WITH PROGRAMMING ROBOT EMIL

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This research aims to explore the synergy between internal records of representation and problem-solving practice while fourth graders use Robot Emil to solve a mathematics-related task. The participants are three pairs of pupils from a primary school in Norway. The data comes from pupils' video-recorded group work when they discuss the solution to a task in a mathematics lesson. We approach data with a specific lens of the problem-solving aspect of computational thinking with an internal record of representation (characteristic) of the task context. The analysis revealed that the internal record of representation in the Robot Emil environment had a catalyst role in conveying pupils to devise a path, evaluating the completed steps and re-devising a path to complete the provided task.

Keywords: Computational thinking, internal record of representation, problem-solving, Robot Emil.

INTRODUCTION

The synergy between using computational thinking (CT) tools as a mediational practice to promote mathematical thinking and developing 21st-century skills – such as *computing* – has opened a door to a new plea. Linking CT practices to learning mathematics has been one of the major interests of educators since there exists an interrelationship between CT, programming skills and mathematical thinking (Hoyles & Noss, 2020; Kalaš et al., 2022; Stephens & Kadijevich, 2020). Consequently, CT as such a new digital literacy has motivated many countries to refresh their curriculums and policies. Regarding classroom practice, *Scratch* (http://scratch.mit.edu), has been a practical (block-programming) tool in current research in primary-level mathematics (Benton et al., 2017; Bråting & Kilhamn, 2021). Similar to other countries (like England and Sweden), in Norway, recently, CT and programming have been regarded as emerging notions in primary and lower secondary school curricula as well (Utdanningsdirektoratet, 2019). In our present project, we focus on a different platform than the Scratch; Robot Emil environment (which will be detailed in the next section).

The background and design of Robot Emil not only provide a context for learning the basics of programming and computing for primary school pupils (Blaho et al., 2019), but also promote conjecturing, exploration, and communication regarding primary mathematics (Turgut et al., 2022). The tasks in the Robot Emil environment were designed according to different dimensions of representation and different levels of control (for a complete discussion, see Kalaš et al., 2018). For example, the environment could provide an internal record of the clicking that could motivate pupils to see the completed steps on the one hand and to estimate further steps to proceed on the other. In the present paper, we focus on the representation of an *internal record* and the problem-solving aspect of CT (Kallia et al., 2021) while fourth graders use programming Robot Emil to solve a mathematics-related task.

PROGRAMMING ROBOT EMIL

Robot Emil is a programming environment which is a method for teaching and learning programming and computing for grades 3 and 4 (Blaho et al., 2019). Indeed, Robot Emil is a combination of unplugged (through a workbook) and plugged activities (through an app on a smartphone, tablet or computer, and workbook). In the present research, we worked with Robot Emil for grade 3, which consists of three worlds. Every world has different levels or units categorised by (eight) letters from A to H. The units have a progression in the tasks and elements of programming and computing skills are implemented for the pupils to discover themselves. Robot Emil can be controlled by clicking (or tapping) on the squares on the stage, but Emil only moves *vertically* or *horizontally* and collects everything on his way. In the first world, Emil collects different kinds of elements, such as fruits, coins, or letters into a tray to show what has been collected. Figure 1 provides an exemplary task G4 (the fourth task in unit G) from the first world.

Help Emil to buy (1) *two buttons for 10 kroner*[1]*,* (2) *three buttons for 6 kroner, and* (3) *four buttons for 11 kroner. Prepare two similar tasks for your learning partner.*

(a) The initial look of G4 (screenshot from App) (b) Appearance of G4 after three clicking/moves

Figure 1. Task G4 from the first world of Robot Emil

In unit G of the first world, a specific function appears as in G4 (Figure 1a), number strips that are called "*bookmarks*". This function limits the number of clicking/tapping. For example, in G4, five clicking (i.e., five movements of Emil) are possible. The tasks guide pupils (who work in pairs) to collaborate, communicate and explore the proposed situations together. Pupils need to discuss and write down their solutions in the workbook. We note that the environment does not give any feedback when pupils are doing the tasks or when they are finished with a task. Therefore, the teacher needs to consider a classroom discussion to ensure shared understanding and meaning (Kalaš et al., 2022).

CONCEPTUAL FRAMEWORK

CT in Mathematics Education

CT, which is now being considered as an umbrella term (Kallia et al., 2021) for several sophisticated skillsets, goes back to Papert's (1980) seminal book about Turtle geometry and LOGO language. Papert's (1980) idea was to create dialectics between programming and cognitive development regarding mathematical learning. Later, Wing's (2006) definition went beyond and underlined the role of CT in different levels of education by stating CT '… just like reading, writing, and arithmetic, should be added to every child's analytical ability' (p. 33). As a consequence, CT has been a part of (new) digital literacy (Stephens & Kadijevich, 2020). There

[1] The *krone*, plural *kroner* is the Norwegian currency, with coins of value 1, 5, 10 or 20. "10-kroning" is a usual way to name the 10-kroner coin.

have been many attempts to describe components/dimensions and aspects of CT in mathematics education (see Hoyles & Noss, 2020 for a broad discussion regarding components of CT). Recently, a particular characterization of CT in mathematics has been provided by Kallia et al. (2021) by highlighting three aspects (p. 179):

1) *problem-solving*: like understanding a problem, hypothesizing a solution strategy and performing the strategy,
2) *cognitive processes*: like abstraction, decomposition, pattern recognition, algorithmic thinking and evaluation of solutions and strategies,
3) *transposition*: like phrasing the solution of a task in such a way that it can be transferred to another human and/or machine.

Kallia et al. (2021) note that these aspects do not necessarily address a need for referring to all CT practices in designing educational settings. From an analytical lens point of view, analogously, we only focus on the *problem-solving* aspect of CT because the first world of Robot Emil environment (as described above) is a very early stage of programming (Kalaš et al., 2018). We limit ourselves to the problem-solving aspect of CT and reformulate its description by incorporating the Robot Emil environment as in Table 1.

Table 1. Description of Problem-solving Steps in the (first world of) Robot Emil Context

Steps	*Description*
Understanding the problem	This step involves understanding the proposed task, like thinking inputs and outputs and how these can be obtained.
Devising and performing a solution path	This step refers to envisaging a solution path and/or a computing strategy and performing it by controlling Emil.
Evaluating the (*solution*) *process*	This step involves evaluating the process, pupils collaborate, discuss and assess their progress by revisiting the screen and workbooks.

As a consequence, we focus on the interrelation between the internal record of representation (dimension) and problem-solving practice as described in Table 1.

Dimension of Representation in Robot Emil Environment

Kalaš and colleagues (Kalaš, 2016; Kalaš et al., 2018), regarding the design of Robot Emil, focus on the cognitive complexity of different levels of control and dimension of representation. Limiting ourselves to the first world and G4 task, we have indirect manipulation as a level of control and representation of the internal record. The first refers to clicking on a certain position to give a signal to Emil to move/float there. For example, by clicking on one button (positioned in the middle of the first row of the square in Figure 1a), Emil will float there and, on his way, will collect two buttons and one 10-kroning. The latter, an *internal record of representation* refers to representing the performed clicking/tapping. Each click turns the square into grey and numbered and greyed positions cannot be clicked again. Additionally, the maximum number of clicks is visible as *bookmarks* as in the initial look (Figure 1a) and on the clicked squares (Figure 1b). In other words, such a representation of the internal record encourages pupils to think in advance and estimate a path to proceed, and it also opens a door to testing the estimated path. This process refers to the second line of Table 1 and motivates us to formulate our research question: *What is the synergy between an internal record of representation and problem-solving practice while fourth graders use Robot Emil to solve a mathematics-related task*?

METHODS

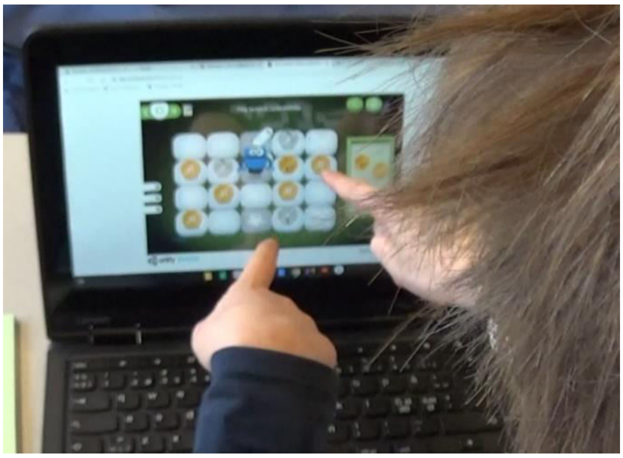
Our case is extracted from piloting Robot Emil in a Norwegian primary school project. We video-recorded three pairs of pupils when they used Robot Emil to solve task G4. Three pairs were grouped by the teacher according to their communication skills and mathematics performance. We focus on a piece of data reported by Turgut et al. (2022) and combine it with other existing data sets. The participants, pseudonyms are Per and Ane, Ida and Mia, and Lise and Hanne were fourth-grade pupils when the data was collected in November 2020.

Task G4, in our previous report, was formulated as an optimisation task due to the dimension of representation. To describe Robot Emil's positions and movements within the context of this paper, we use an imaginary coordinate system where the first number represents the row (from top to down) and the second represents the column (from left to right). For example, in Figure 1a, Emil is located at [4,3]. Following the internal record of representation in Figure 1b, Emil floated this path: [4,4] → [4,4] → [2,4] → [2,2]. We note that questions (1), (2) and (3) of the G4 task are independent of each other. The data includes video recordings of pupils' work with researcher/observation notes. The videos were later transcribed and translated from Norwegian into English by researchers. We discussed an initial analysis of the data with a particular reference to the problem-solving aspect of CT. This was later combined through the lens of representation of internal record.

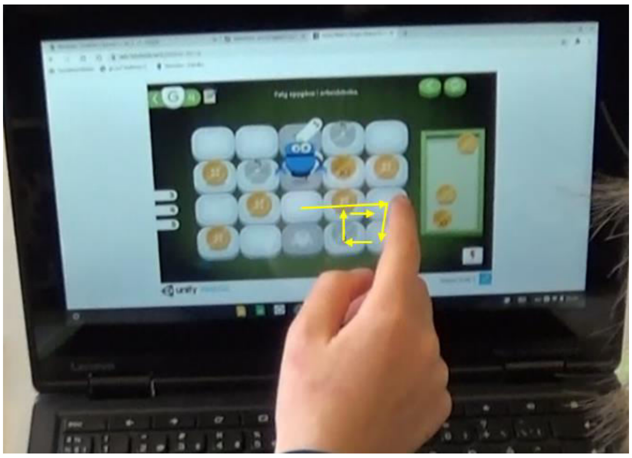
FINDINGS

The Case of Per and Ane

Per and Ane started to discuss the first question of the task. In the beginning, due to bookmarks provided in the task, Per thought that they had five possible clicking and should collect as many buttons as possible. Later, Ane read the task again and immediately devised a possible path by tracing her pencil on the screen: [4,3] → [4,1] → [3,1] → [3,2]. Then, Ane expressed to collect 10-kroning on [3,3], which was a correct solution without performing the devised path. At this moment, Per suggested an optimized solution by clicking on [1,3] and stated '*It is 10 kroner and two buttons*'. After having two solutions, Ane explored what to do with the remaining moves (pointing on the screen with a pencil, at bookmarks for moves on the left-hand side), and they focused on the third question of the task (4 buttons for 11 kroner). Ane refreshed the screen and devised a path [4,3] → [1,3] → [3,3] → [3,5] by pointing at the screen and Per noted the steps and suggested additional movements by clicking on [2,3] and pointing at [2,5] (Figure 2a).



(a) Per's suggestion to proceed



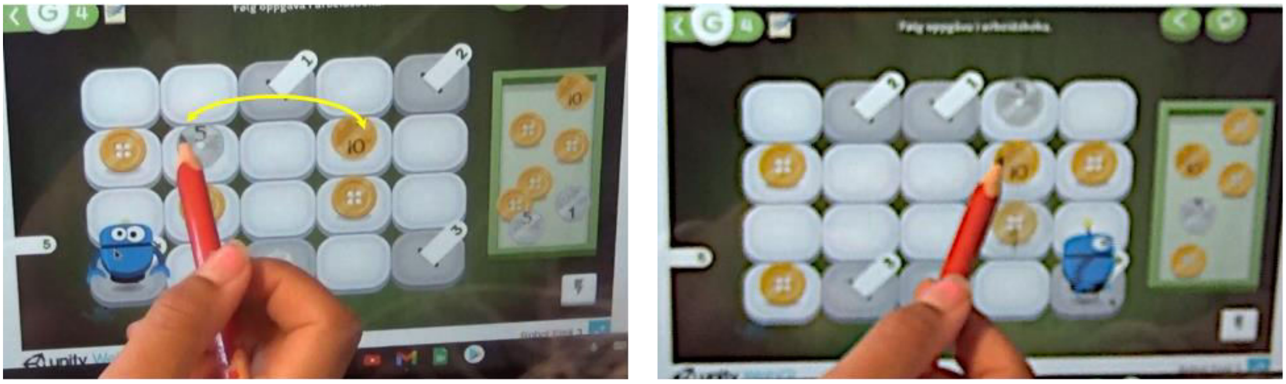
(b) Ane's (an)other devised path

Figure 2. Per and Ane's discussion to complete the task

Ane tried collecting more buttons by having a plan thanks to her experience coming from the initial part of the task, but she realized, however, she could not, because they could not click on a position where Emil initially was (i.e., greyed square) (Figure 2b). Then, pupils discussed the situation; Per suggested focusing on four buttons for 20 kroner since two buttons cost 10 kroner. Ane reminded him that four buttons are 11 kroner at first, later she moved to the second question of the task by stating '*or maybe three buttons which are six kroner? If we go …*'. Ane devised a path and performed it by [4,5] → [1,5] → [1,3]. Two buttons and 6 kroner were in the tray. Per reacted that something was missing and Ane clicked on [2,3] to collect one more button, which completed the second question of the task. Most likely due to one remaining bookmark, Per expressed '*it should be possible to get one button*'.

The Case of Ida and Mia

Ida and Mia started to solve the third question of the G4 task (4 buttons for 11 kroner), as based on the marks in their workbooks, they already solved the first two questions. Emil was on [1,3] and after Mia's click, he floated to [1,5], so there were 2 buttons and 15 kroner in the tray. Then Mia instructed him to move to [4,5] and Ida tried to devise the next move by pointing with a pencil at [4,4]. However, Mia performed her strategy and clicked on [4,1]. At this moment 4 buttons were collected, and the girls started to plan how to collect 11 kroner. Ida stated: '*And so we need eleven kroner, but we have fifteen ... five here ... and this* /she pointed with a pencil at 5 and 10 on [2,2] and [2,4]/ *will be fifteen*' (Figure 3a). Ida and Mia didn't understand the task correctly, as they focused on buttons and coins separately. In the next attempt, they tried a different path [1,3] → [1,2] → [4,2]. Then Mia counted 3 buttons in the tray and Ida suggested to Mia (who was controlling Emil) to instruct Emil to go to [4,5]. Mia noticed they had only one move left, however, Ida suggested going to [2,4] (Figure 3b). Mia expressed the impossibility again: '*Do you think it is possible to go there when we only have one move left?!*'. Ida understood the problem and came up with a new strategy: '*I had an idea! Because if you had gone here* /she pointed with a pencil at [4,4]/, *we could have gone like this* /showed the path [4,4] → [2,4] → [2,5]/'.



(a) Ida counts the left coins on the stage (b) Ida's suggestion to Mia to complete the task

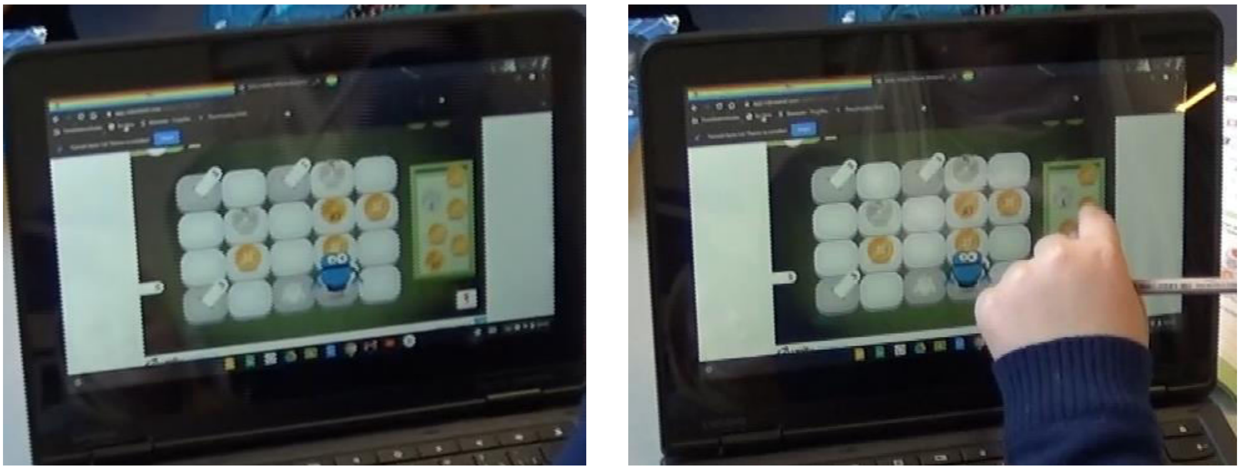
Figure 3. Ida's and Mia's discussion to complete the G4 task

Mia confirmed she understood, and Ida added that this strategy also involves collecting 11 kroner. However, to perform it, they would still need six moves, although the last move to [2,5] is unnecessary because they would have had 4 buttons and (in their way of thinking) 11 kroner if Emil got to [2,4]. After refreshing the screen, Mia instructed Emil to perform the path [1,3] → [1,2] → [4,2] → [4,4] and then the girls discussed the last move, whether it should have been to [1,4] or

[2,4]. At last, Mia sent Emil to [1,4] and Ida confirmed, by counting in the tray, that they collected 4 buttons. Then Mia started to count the value of all coins in the tray, yet Ida convinced her that the solution was correct: '*Look here! Eleven.* /she pointed with a pencil at 10 and 1 coins, back and forth three times/ *So then we have eleven kroner! I think we managed it.*' After accepting the solution, the girls drew it in the workbook. Despite the fact that the internal record of the process was on the screen, the girls tried to recall from memory which path Emil followed.

The Case of Lise and Hanne

Lise and Hanne completed three questions in G4 as they noted in their workbooks. For example, they performed a path [1,3] → [1,1] → [4,1] → [4,4] to complete the third question (four buttons for 11 kroner) (Figure 4a). We note that Lise and Hanne discussed different possibilities more in the last part of the task, which was about preparing two similar questions for each other. Lise proposed two subtasks, "seven buttons for 1 krone" and "two buttons for 15 kroner" for Hanne. Before doing this, Lise checked these questions by counting the buttons on the screen (Figure 4b). We observe that Lise gave the first task on purpose being aware that it was not possible to solve it because she already has a good understanding of the task situation, most likely thanks to the internal records of representation.



(a) Lise and Hanne's solution for the third question | (b) Lise counts (possible) buttons

Figure 4. Lise and Hanne's solution to the third question in G4 and Lise's counting of buttons

Hanne realised that it was impossible to collect all the buttons without picking up more than 1 krone. This was probably thanks to devising a path in her mind by considering the number of bookmarks. Later, Lise updated the task to "seven buttons for 11 kroner", Hanne refreshed the screen and tried to solve this new task. Hanne started to think and devised a new path on the screen, [1,3] → [1,1] → [4,1] → [4,4]. Later, Hanne pointed at the tray for a while and evaluated the situation, '*this is not possible Lise*' and by explaining further, '*one could only get … one, two, three, four, five …*', she expressed that it was impossible to collect more buttons. Lise replied that she would check it, which means that Lise wanted to think once more about it. Then Lise updated the task to "three buttons for 11 kroner". Hanne easily devised a path and completed the task. Further, regarding "two buttons for 15 kroner", Lise stated '*this is actually very easy*', which showed us Lise had – at least – one solution in her mind. Hanne refreshed the screen and solved it easily by [1,3] → [1,4].

Lise solved Hanne's first task (four buttons for 15 kroner) by instructing Emil to perform five moves [1,3] → [1,2] → [3,2] → [3,1] → [2,1]. Later, Hanne asked for "three buttons for 10 kroner”. Lise tried to solve this by tracing a devised path, [3,3] → [4,3] → [3,3]. Lise changed her mind, possibly due to greyed squares which limit her to proceed. Later, Lise devised a new path and performed [4,1] → [1,1] → [1,3], then waited and pointed on [4,2] and [3,3]. Then Lise evaluated her finding and stated, ‘*it is not possible*’. So, we conclude that Lise overviewed the internal record on the screen and realised that in this situation it was impossible to collect the targeted amount without picking up more buttons. After encouragement from Hanne, Lise devised a new path and instructed Emil to float the path [2,3] → [1,3] → [1,1] → [1,2].

CONCLUSIONS AND DISCUSSION

In this paper, we focused on the synergy between an internal record of representation in the Robot Emil environment and fourth graders’ problem-solving practice. Our overall conclusion is that the internal record of representation had a catalyst role in pupils’ understanding of the task, devising, refreshing, and performing a path, evaluating the completed steps and re-devising a path to proceed. We observed three main components of the internal record of representation: *bookmarks*, *greyed squares* (Kalaš et al., 2018), and (we have added here) the *tray*. Because, in our case, pupils revisited the tray a few times to count the collected items and compare these with the remaining bookmarks. The items in the tray cannot be removed and the tray displays what has been collected synchronously. Through Figure 5, we overview pupils’ problem-solving practice from the lens of the internal record of representation.

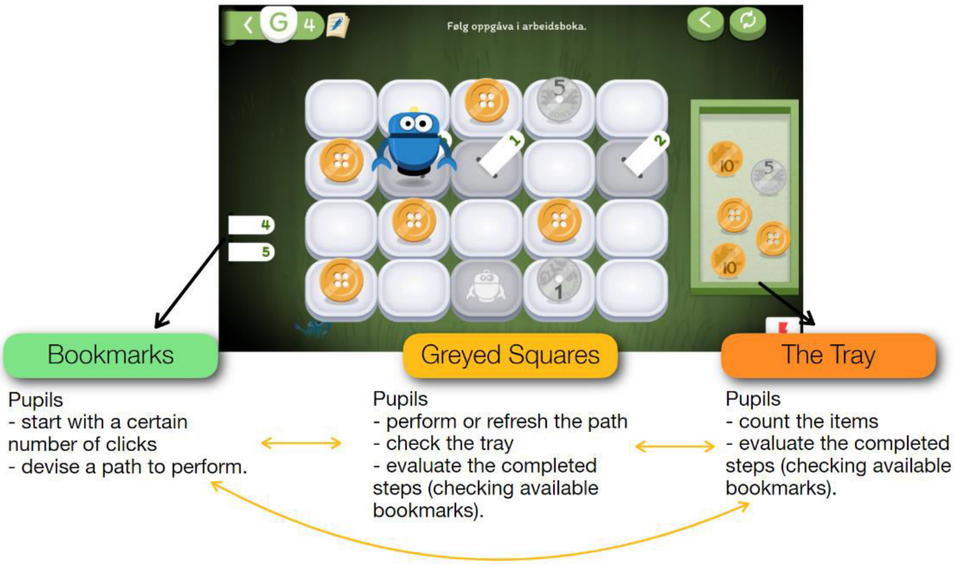


Figure 5. Summary of fourth graders’ problem-solving practice from the lens of internal record

Figure 5 summarises that, in our case, the synergy between bookmarks, greyed squares and the tray conveyed pupils’ problem-solving, like starting with a certain number of clicks, considering this and devising a path. This follows performing or refreshing the path and checking the tray. The next step is evaluating the completed steps (checking available bookmarks), counting the items in the tray and refreshing the screen if needed.

We note that tasks in the Robot Emil environment are getting more complex. For example, in the preceding worlds, there exist the ideas of direct and *computational control* (Kalaš et al., 2018), where the user needs to program Emil by considering future moves and actions before Emil carries out these steps. These tasks are relevant to explore a number of specific CT components, like pattern recognition and generalisation (Hoyles & Noss, 2020) and algorithmic thinking (forms of different mathematical reasoning, like spatial or geometric reasoning) (Stephens & Kadijevich, 2020). Therefore, there is a need for analysing the interrelationship between, for example, spatial reasoning and pattern recognition and developing, testing, and refining algorithms through Robot Emil in future settings.

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