

USING GRAPHING CALCULATORS IN TEACHER EDUCATION

Francesco Saverio Tortoriello and Ilaria Veronesi

University of Salerno (Italy); fstortoriello@unisa.it, iveronesi@unisa.it

This paper focuses on the use of graphing calculators in a mathematics laboratory where teachers become familiar with this kind of technology. Twenty in-service high-school teachers were involved in didactic activities on successions and limits of succession through the use of calculators. A qualitative analysis was developed, teachers were interviewed in focus groups and semi-structured questionnaires and data was collected from a qualitative point of view.

Keywords: Graphing calculators, technologies in math education, constructivist approach, integrated laboratory

INTRODUCTION

In 2017, the Italian Minister of Education amended the legislation to allow the use of GC during the final examination in scientific high schools (Ministerial Ordinance no. 257/2017, Article 18). This amendment has generated increased interest in this technology and prompted manufacturers to develop training courses. However, it has yet to significantly alter the way teachers approach mathematics and its models with such a tool. Previously, the use of GC in teaching activities was prohibited, likely due to their use in course final exams, thus, few works had been produced on the subject up to that point and in Italy there is no significant relevance on its use. At an international level, on the other hand, GC is a teaching tool used by many school systems for almost 40 years and has been the subject of research in teaching.

This paragraph will provide an overview of experimental results, interpretations, and conclusions of educational activities developed with GC. We are able to review over four decades' worth of experiments and models conducted in various school levels and in relation to various mathematical topics. In 1996, Penglase and Arnold (Penglase and Arnold, 1996) conducted an extensive analysis of the published works on GC and presented an in-depth analysis of educational levels, activities, structures, and instructional outcomes, with a focus on their potential for mathematics teaching from both curricular and pedagogical perspectives (Tnan, 2011).

In the coming years, numerous studies have been conducted on the topic of GC, with specific focus on their impact on students' and teachers' behavior (Nzuki, 2016; Parrot, 2014 and 2018; Mc Culloch, 2009; Leng, 2011, Quesada, 1994 and 2008; Ross, 2017; Mitchelmore 2000; Asli Ozgun-Koca, 2009; Tan, 2015; Lee, 2005). Furthermore, a number of authors have examined the process of teaching, learning and the formation of mathematical understanding via these technologies (Burril, 1996 and 2002; Drijvers, 1996; Ellington, 2003; Trouche, 2010; Robutti, 2010), as well as teachers' attitudes towards GC (Bologna et al. 2019 and 2021; Tortoriello and Veronesi, 2022). Generally, these papers describe educational activities conducted on a small group of learners, typically a class, and typically centering on a single topic.

The Mathematics Education Research Group of the Department of Mathematics at the University of Salerno has been working for years on a plan for educational experiments that add to the students' extracurricular activities and sharpen their critical thinking capacity through the introduction of interdisciplinary topics that enrich their knowledge of Mathematics and its role in connecting classical and scientific fields (Rogora and Tortoriello, 2021). This plan also seeks to enhance the students' skills by providing them with the latest technological tools to interpret a rapidly changing reality (Tortoriello and Veronesi, 2021 and 2021).

In this paper in particular we decided to present and discuss a teacher education case, and specifically the activity of a lesson dedicated to successions, to see how appropriate teachers consider the use of new technologies for teaching mathematics.

There is a need for practical analytical research tools and frameworks that offer the potential to analyze teachers' technology integration in mathematics. A commonly used framework, derived from Shulman's (1986) pedagogical content knowledge, is Koehler and Mishra's (2009) Technology Pedagogical and Content Knowledge (TPACK), which focuses on the aspects of teacher knowledge that are needed for the effective use of technology in the classroom.

The aim of this study is to assess how teachers consider it appropriate/effective to integrate technologies into the pedagogical and methodological aspects of mathematics. Through the lens of the Mishra framework, a training on the use of GC technology has been designed to improve teaching so that the technologies are integrated into teaching and disciplinary skills at the intersection of TPACK's knowledge domains. The activities carried out have been structured with the aim of acting as an example and stimulating teachers to reproduce the same teaching model.

In this study, a qualitative analysis was conducted because we wanted to record and analyze the impressions of teachers through observation during activities, focus groups and semi-structured questionnaires administered to teachers at the end of the activities.

MEANS AND TOOLS FOR ANALYSIS

This paper addresses issues of integration of theory and practice in the maths education and describes a laboratory that, beginning with episodes from real-life problems, proposes a set of theoretical models and situates these models in examples. The educational project focuses on the aim of creating and implementing activities that are tailored to the teachers' needs of developing the students' curricula through the GC to be used to facilitate learning. The project involves researchers who are knowledgeable in the fields of education, curriculum, and instruction and provides guidance to teachers in the development of activities to support the learning process. The activities are designed to promote engagement and collaboration between teachers and focuses on developing skills and knowledge that are essential for success in the classroom. Through this process, teachers are able to gain valuable insight into how to effectively utilize new technology, learning strategies, and content materials in the classroom with the aim of benefiting students from engaging with the new content in a meaningful way.

The aim of the research was to pay attention to the experiential data emerging in the performance of the activities using the research techniques of observation of free and semi-structured interviews (in the form of questionnaires) and focus groups. For the qualitative analysis of data through the lens of TPACK, a positive attitude towards the GC tool seems to emerge, the use of GC designed with a teaching perspective, the acquisition of new technological skills.

METHODS

The modality that appears most suitable, as it is easily repeatable in the classroom and therefore effective for the acquisition of technological skills by teachers, is the one that sees the domains of knowledge integrated and exemplified through modeling by the researcher-trainer and put into practice in "authentic" contexts by the participating teachers.

Contents and participants

In order to provide participants with didactic tools immediately usable in the classroom, prior to the beginning of the course the themes to be developed were agreed with them and two topics of

interest were identified: the approach to the analysis and the problems of choice. The activity presented in the paper concerns the first theme and focuses on the concept of succession.

The participants in the educational workshop are four teachers of scientific high schools who teach in the fourth class, penultimate year of the course of study.

The laboratory of graphing calculators

Given the importance of the changes made by the Minister of Education to the final exams of scientific schools and the positive educational impact that the use of GC can have in terms of skills acquired by students, the research group designed an experimental laboratory. In the activities, teachers worked together in group to gain a deeper understanding of mathematical objects by viewing them in a variety of ways through the GC menu with different approaches, numerical, analytical, graphical, tabular. This provides an effective learning experience and allows participants to gain a more comprehensive understanding of the strategies that can be used in teaching.

Data collection and data analysis

As specified in the introduction, the analysis of the research was qualitative and developed in three different areas: the administration of questionnaires, the organization of focus groups and the interview of the participating teachers.

The questionnaires prepared are semi-structured and are divided into two parts, one with multiple answers and the other with open questions. Multiple choice questions concern the organization of the course in its entirety. We investigated the level of difficulty, the commitment and the workload required to follow the course, the organization with reference to the contents and methods and asked for an evaluation of the didactic impact in their teaching. It was asked to express an evaluation on the choice of contents addressed, on the clarity and organization, on the achievement of the learning objectives. Another group of multiple choice questions investigates the behavior of the course participants and the interaction between trainer and teachers and between the teachers themselves, the climate established during the activities, the stimulated interest and the full use of the paths by everyone. In the open question part, the teachers were asked what were the most useful aspects of the course from their point of view, what were the strengths and weaknesses of the course and they were asked to propose improvements for this type of activity to be repeated in the future.

In the interviews the trainer met each of the four participating teachers and asked them to evaluate the skills acquired with respect to the use of GC and the design of a mathematics didactics implemented with technology.

In the focus group (Zammuner, 2003) the researcher acted as a facilitator, directing the conversation towards the GC topic to garner the teachers' perspectives through questioning that guided the discussion. Group interviews proved to be an effective method in gathering data efficiently due to the interactivity between participants.

The activity

The activity took place in four meetings held during extracurricular hours. Using a heuristic approach, the participants explored the functionality of GC by engaging in simple content exercises. The topics were already known to the participants, allowing for an easy transition into the exploration process. Through this method, the group was able to dive deeper into the functionality of GC and further their understanding. After becoming familiar with the various menus of the GC, that of calculation, graph, tables, sequences, the activities were oriented to solve reality problems.

The activity we present in the paper refers to the study of sequences both as functions in $n \in \mathbb{N}$ (figure 1) and as recursive functions (figure 2).

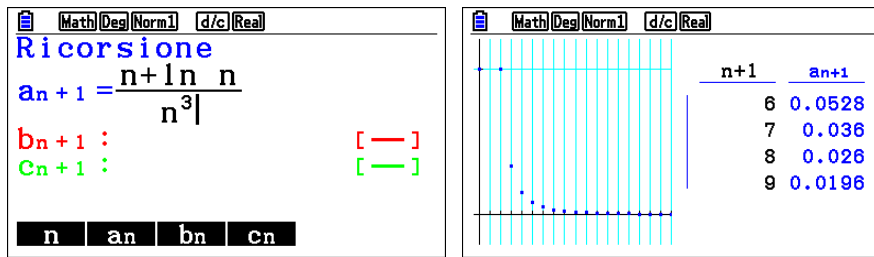


Figure 1. An example of the activity on recursion

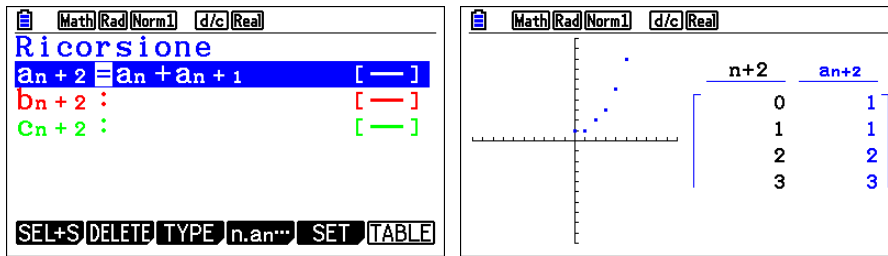


Figure 2. The recursive Fibonacci sequence

Using the recursion menu, the behavior of a sequence was analyzed both from a tabular and graphical point of view and it was possible to compare the two models on the same screen to deduce the behavior of the sequence. The visualization of the trend of a sequence allowed to introduce the concept of limit by observing it both from the point of view of values calculated in the table and as a sequence of points on a graph with behaviors that depended on the defined mathematical relationship. The teachers felt that the meaning of convergent, divergent and indeterminate sequence can be easily translated into the observations of the behavior of the graph or table.

THEORETICAL FRAMEWORK

The Italian school reality has not so far officially provided for the adoption of the TPACK model, a model that outlines the knowledge domains underpinning teaching/learning processes in which technology plays a substantial role and, as a conceptual reference framework in the definition of initial and continuous teacher training courses. However, the set of regulatory interventions that have taken place over time defines a theoretical and methodological horizon that is in fact compatible with the model itself. Technology is changing learning and teaching strategies and, among the many questions that this introduction raises, the one that investigates what knowledge a teacher must have to successfully manage a teaching experience that involves the use of technologies in a decisive way, occupies a central role. The well-known TPACK model proposes an answer to this question, explaining that a teacher must be competent with respect to the intersection of three types of knowledge: pedagogy, disciplinary content and technology.

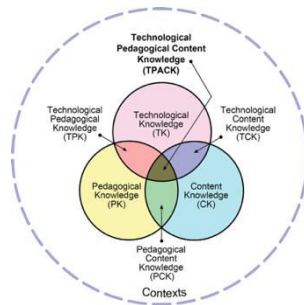


Figure 3. The TPACK model, © 2012 by tpack.org.

The teacher must therefore be competent not so much and not only in relation to the technology itself, to the pedagogy and to the specific content of his disciplinary field, but above all in relation to the intersections of these domains, or to those uses of technology that support appropriate pedagogical strategies in relation to his subject of teaching.

RESULTS AND DISCUSSION

Analysis of questionnaire responses led to the following results.

The teachers were asked to express with a vote from 1 (totally disagree) to 5 (totally agree) on the following aspects of the course:

- The activities were clear and organized, 100% voted 5
- experts stimulated participants' interest, 50% voted 5 and 50% voted 4
- created a positive atmosphere during the lessons, 100% voted 5
- Learning objectives were achieved, 25% voted 5 and 75% voted 4
- course content was organized and well planned, 100% rated 5
- course workload was appropriate, 25% voted 5 and 75% voted 4
- The course was organized to allow everyone to participate fully, 100% voted 5

Teachers were asked to express with a vote from 1 (very negative) to 5 (excellent), on the following aspects of the course:

- The level of commitment required by the course, 100% voted 3 (adequate)
- the organization with reference to the contents and methods of the course, 100% voted 5
- usefulness of course activities to improve academic performance, 100% voted 4
- the level of difficulty in participation, 50% voted 5 and 50% voted 4
- the interaction between the teacher of the course and the students, 100% voted 5
- the choice of topics addressed, 100% voted 5

To the open question on which aspects of this laboratory activity were most useful, the teachers replied that they had learned to use the graphing calculator and understood its potential especially from a didactic point of view. When asked to briefly describe a strong point of the course, the teachers answered the expendability and repeatability in class of the activities carried out and the acquisition of a method to make the students discover the calculator with an exploratory approach. No weaknesses of the course emerged. When asked how they suggested improving the workshop they had done, the teachers indicated that it might be useful to have a greater number of meetings to develop more different topics.

In interviews with teachers it emerged that they had never previously used the graphing calculator to carry out teaching activities in the classroom as they did not know the tool and therefore did not feel masters of the potential to be able to replicate and teach them to students, as they did not want to be unprepared in front of questions on instructions or use to which they could not give answers. They expressed their concerns earlier in the course about the risk of students relying entirely on the results of the calculator by not developing critical thinking and stated that they changed their opinion during the course because, using it, they understood that the calculator can help students observe and deduce properties but does not replace the study of mathematics that remains necessary. It has also emerged how GC can be a valuable teaching aid for students who have learning difficulties.

In the focus groups we discussed the theoretical, pedagogical and methodological effectiveness of the GC focusing on the strengths and weaknesses of a teaching in which students use technologies and on the engagement of students in laboratory activities with GC instead of classic math lectures.

The teachers highlighted that GC can be particularly effective for an approach to mathematical contents according to the various learning styles, especially for those areas in which there are "dynamic" trends in the study, such as in sequences in which graphic and tabular observation helps to understand the behavior of the sequence and introduces in a natural way to the concept of limit. The teachers also underlined how the natural teaching with the GC develops in small groups as it happened during the laboratory and how they think they can replicate it in the classroom as in a Peer Education collaboration helps students to overcome mutual difficulties in a comparison also thanks to their natural predisposition to the use of technological tools.

CONCLUSIONS

For the qualitative analysis of data through the lens of TPACK, a positive attitude towards the GC emerges, in fact teachers in the interviews and in the focus groups sublined positively the use of GC designed with a teaching perspective and the acquisition of new technological skills useful to project classroom activities tailored on students needs.

With reference to the TPACK Theoretical Framework, the teachers and participants in the laboratory activities understood how the component of technological knowledge of the GC is not an end in itself but intervenes and collaborates in relation to the pedagogical and content area.

During the course the teachers have experienced that their competence in technology must not simply be a validation of experience and skills but must be integrated and must be at the service of the disciplinary contents that are proposed and the pedagogical tools chosen to develop them in the classroom. Technology therefore becomes an adequate support for pedagogical strategies and theoretical insights.

From the participants' comments emerged that for teachers the GC is an effective tool to promote conscious exercise and convey new mathematical contents bypassing the mere calculation, being able to focus exclusively on theoretical development and exploration. The teachers, who had never used GC before their participation to the project, underlined the positive impact in terms of active participation and recognized the didactic effectiveness of the tool GC that can promote and empower laboratory activities of teaching mathematics. Furthermore, the activities can help to build skills such as problem-solving, teamwork and creativity, which can all be beneficial in other aspects of life.

The experimental laboratory enabled the participants to actively engage with the material, as well as to practice their problem-solving and analytical skills. The interaction between participants and knowledge is mediated by the GC tool, providing an innovative way to explore mathematical objects through a variety of stylistic approaches, allowing for a heuristic exploration of mathematical concepts. In addition, this tool provides a platform for participants to better understand the abstract concepts that form the basis of mathematical knowledge.

Ultimately emerged that engaging in these activities can help to create a more meaningful and favorable approach also for students with learning difficulties.

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