UNPACKING TEACHER BELIEFS ABOUT MATHEMATICAL DIGITAL COMPETENCY

Daniel Thurm[1], Eirini Geraniou[2] and Uffe Thomas Jankvist[3]

[1]*University of Siegen, Germany; daniel.thurm@uni-siegen.de*

[2]*University College London, United Kingdom; e.geraniou@ucl.ac.uk*

[3]*Aarhus University, Danish School of Education, Denmark; utj@edu.au.dk*

The recently proposed construct of mathematical digital competency (MDC) entails that student understanding of mathematical concepts may be almost inseparable from digital technology (DT) use, and that students might only be able to "do" mathematics with DT. We report on a case-study in which we investigated how two German pre-service-teachers (PSTs) think about the connected notion of MDC, and discuss their opposing views. We found that the belief that "mathematical understanding" is reflected by being able to explain and do mathematics without DT was a fundamental reason to oppose the notion of MDC for one of the two PSTs. In contrast, the other PST supported the notion of MDC by placing the DT in relation to other tools. We conclude that beliefs about MDC are likely a critical part of teachers' own MDC.

Keywords: mathematical digital competency, digital technology, teacher beliefs, predicative knowledge, operational knowledge

INTRODUCTION

Our research work entails the investigation of a somewhat novel dimension of teacher beliefs in the context of teaching mathematics with digital technology (DT). The starting point for the research study was the work of Geraniou and Jankvist (2019), who argue that mathematical competencies and digital competencies are rarely seen as a connected whole, even though students will have to simultaneously activate and use these competencies. They thus conceptualize the construct of "mathematical digital competency" (MDC), describing an amalgam of mathematical and digital competencies. They show that such an amalgam entails that a student's understanding of a mathematical concept may almost inseparably be connected to DT and the student's instrumented techniques. In this paper, we investigate PSTs' beliefs about such an amalgam, how they justify their beliefs, and how their beliefs are related to other beliefs about the potentials of DT. Throughout the paper the term "digital technology" (DT) refers to mathematics-specific DT such as function plotters, dynamic geometry systems, computer algebra systems and multi-representational tools.

THEORETICAL BACKGROUND

Mathematical competency can be defined as "someone's insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations" (Niss & Højgaard, 2019, p.14), while digital competency has been conceptualized as "the set of knowledge, skills and attitudes […] that are required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge" (Ferrari, 2012, p.43).

Geraniou and Jankvist (2019) linked both kinds of competencies by using the theory of instrumental genesis (TIG) and the theory of conceptual fields (TCF). TIG (e.g., Guin & Trouche, 1999) describes the process of transforming DT (an artefact) into a mathematical instrument, which is a psychological construct that combines (parts of) the artefact and cognitive schemes in which technical knowledge about the artefact and mathematical knowledge are intertwined. TCF (Vergnaud, 2009) highlights that a concept comprises a set of schemes, a set of situations and a set of linguistic and symbolic tools of representation, and that different concepts and situations are interconnected forming conceptual fields. The set of situations gives meaning to the concept and acts as a point of reference. TCF also emphasizes the distinction between operational knowledge that makes it possible to do something and predicative knowledge that makes it possible to describe and give reasons.

Geraniou and Jankvist (2019) use TIG and TCF to investigate the simultaneous activation and development of mathematical and digital competency, which they call "mathematical digital competency" (MDC), and highlight, that the situations that make up students' conceptual fields "[…] may be embedded so deeply in a techno-mathematical discourse that, potentially, also their understanding of the mathematical concepts involved is almost inseparable from the digital tools and the students' instrumented techniques" (p. 42). This could for example mean that the set of situations that students use as points of reference to give meaning to a concept will largely comprise situations involving DT. Hence, a student's predicative and operational form of knowledge will be inherently intertwined with DT, which means that a student might only be able to think about, describe and explain mathematics with reference to a DT (predicative knowledge) and do mathematics with DT (operational knowledge).

Continuing the work on the MDC framework entails to investigate what teachers believe about such a potentially close interwovenness as a first step in convincing teachers for the value of supporting students developing MDC. This very argument is aligned with Tabach's plenary talk at PME44 (Tabach, 2021), in which she pointed out that besides MDC for students the research community must try to provide a parallel conceptualization of MDC for teachers. We propose that such a parallel conceptualization comprises teachers' beliefs about MDC. Teacher beliefs – which can be defined as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259) – are particularly important, since they act as a bridge between knowledge and action and hence may have a profound impact on classroom practice (Thurm & Barzel, 2020; 2021). Clearly, a teacher can have different belief positions with respect to the relation between mathematical and digital competencies. On one extreme, one can fully embrace the "*connected position*" of MDC (e.g., that students' understanding of the mathematical concepts involved may be almost inseparable from DT). On the other extreme, one could strongly favour an "*independent position*" believing that mathematical and digital competencies should be clearly separated (e.g., a student should be able to think about, explain and do mathematics without DT).

In a related quantitative study (Thurm et al., 2022), with n=198 PSTs from three German universities, the authors of this paper showed that many PSTs oppose a "*connected position*" as conceptualized by MDC and strongly favour an *"independent position"* instead. This can be problematic, if the goal is to develop students' MDC. However, investigating teachers' belief position (e.g., "*connected position*" vs. "*independent position*") should be augmented with investigating teachers' *belief argumentations* – i.e., how teachers reason for their position (Rott, 2021). Furthermore, it is important to consider teachers' *belief system* (Green, 1971; Philipp, 2007) in which beliefs form clusters, since beliefs "come always in sets or groups, never in complete independence of one another" (Green, 1971, p. 41). Beliefs within a teachers' belief system may be logically connected. In addition, some beliefs may be more central than others, while some beliefs may be inconsistent with one another. The results of Thurm et al. (2021) indicate that PSTs' beliefs about the potentials of DT and beliefs about whether DT should only be used if mathematics is understood without DT, may be central in the context of beliefs about MDC.

RESEARCH QUESTIONS AND METHODOLOGY

The study aims to explore how PSTs justify their beliefs (*belief argumentation)* about MDC and to investigate beliefs about MDC in relation to PSTs' *belief system*:

- RQ1: How do pre-service teachers justify their beliefs about the relation of mathematical and digital competencies? *(Belief argumentation)*
- RQ2: How are beliefs about the relation of mathematical and digital competencies related to beliefs about DT? *(Belief system)*

To answer these research questions, we use the case-study approach (Flyvbjerg, 2006) and investigate two PSTs' beliefs by means of semi-structured interviews. Case studies provide deep insights into phenomena and help to capture the complexity of a phenomenon. In particular, prototypical cases allow to exemplify important features and tensions among teachers' belief systems (e.g., Andrà et al., 2021).

To prepare PSTs for the case-study interview, we constructed an initial "information sheet" that outlined the "independent position" and the "connected position" and illustrated each of these with a concrete example. In a pilot study, 20 PSTs were asked to read an information sheet and subsequently elaborate on the reasons underlying their position. The pilot study indicated that some PSTs had difficulties understanding the difference between the two positions. Furthermore, the PSTs were often focusing on the operative form of knowledge, and several PSTs were quickly side-tracked to talk only about the potentials or risks of DT use. Based on these observations, we revised the information sheet as well as the interview questions. We focused the information sheet on predicative knowledge and extended the illustrating examples. The final information sheet is given in figure 1.

Independent position	**Connected position**
A learner should be able to think about and explain mathematics **independently with DT.**	It is acceptable if a learner can only think about and explain mathematics **in connection with DT.**
This means: ● A learner should be able to explain and think about a mathematical concept or relationship without making references to a DMT	This means: ● It is acceptable if a learner can explain and think about a mathematical concept or relationship only by making references to a DMT.
Examples: - A learner can explain the role of the parameter "a" in $f(x)=ax^2$ without verbally referring to a DT, e.g., the learner says: "If I draw a parabola with a=2, it will be wider than if I draw a parabola with a=3. Therefore, "a" determines the width of the parabola").	Examples: - A learner can explain the role of the parameter "a" in $f(x)=ax^2$ only by verbally referring to a function plotter, e.g., the learner says, "If I take a slider that controls "a" then the width of the parabola would change. Therefore, "a" determines the width of the parabola."
- A teacher asks a student what comes to his mind when he thinks about the derivative. The student can think about the concept independently from DT. The student answers: "I think of tangents to the graph of the function."	- A teacher asks a student what comes to his mind when he thinks about the derivative. The student answers: "I think of a function in GeoGebra where I can move a tangent dynamically along the graph of the function."
- A learner can explain linear functions without making references to DT: "A linear function is a function of the form y=mx+b, where m and b are numbers, for example 3x+5".	– A learner can explain linear functions only by making references to GeoGebra. He says for example: "If I use GeoGebra to vary m and b in y=mx+b then we would get many different examples of linear functions like 3x+5"

Figure 1. Information Sheet

Next, we set up four subsequent interview questions, each clearly addressing one distinct aspect:

- [**beliefs about MDC-predicative**]: "What are the reasons underlying your decision about whether a student should be able to think about, explain and give examples with or without DT?"
- [**beliefs about MDC-operational**]: "To what extent should a student be able to "do" mathematics without DT, e.g., solving equations or drawing graphs?"
- [**beliefs about time point of DT use**]: "How much do you agree with the following statement: "DT should only be used after the mathematics has been thoroughly understood without DT. Please explain the reasons underlying your position.""
- [**beliefs about potentials of DT**]: "Do you believe that DT can help support the learning of mathematical concepts? If yes, why? If no, why not?"

After another pilot in which no further problems arose, the final information sheet (figure 1) was given to 12 mathematics PSTs studying in the 5th semester at a German university. The PSTs were asked to read the sheet carefully and position themselves on a 6-point-scale with 1 being the "*independent position*" and 6 the "*connected position*". Afterwards, two PSTs, one strongly favouring the "*independent position*" and the other "*connected position*", were asked to take part in an interview that addressed the four questions given above. In the following, we describe the two cases of Lara and Pete.

CASE 1: LARA ("INDEPENDENT POSITION")

[beliefs about MDC-predicative]: Lara stressed that if a student can explain, think about, and give examples only with respect to a DT, this would not allow her to see whether the student understands the mathematics or whether (s)he only possesses technical competency:

Lara: I think it's important to make sure that you teach the whole thing without DT, so that you can really make sure that the students understand it.

Moreover, Lara stated that being able to explain and think about mathematics without reference to DT is more "true" mathematics.

[beliefs about MDC-operational]: In the same sense, Lara argued that if a student cannot "do" mathematics without DT, then she cannot be sure that the student understands:

Lara: I see the same again, what I have just said. For example, if I'm supposed to draw something or maybe determine some things, I find that it's not clear whether the students are aware of what they're doing.

Furthermore, she stressed that doing mathematics by hand / pen & paper helps to foster understanding. For example, she argued that a student should always be able to create all geometrical constructions without use of DT.

[beliefs about time point of DT use]: Because of her previous points, Lara clearly believed that DT should only be used when the mathematics is already understood thoroughly (without DT) in order to make sure that the students understand the mathematics, and to be able to check the understanding of the students:

Lara: I think that is exactly what I just said. […] because I think that they should first try it out themselves, or construct it themselves, so that one can see that the children have understood it and have the chance to learn to understand the connection between different things. And when the knowledge is available, you can say now I'm working with DT.

[beliefs about potentials of DT]: Lara was generally positive about DT and had even considered to write her Bachelor thesis about DT. She stressed that DT can be helpful, if knowledge and understanding have already been established. She highlighted that DT offers ways to dynamically modify parabolas, and to easily construct geometric objects. Furthermore, she stresses the potentials to discovery mathematical relations on your own:

Lara: One can experiment, for example. [...] we had drawn a triangle with GeoGebra and drawn the bisector, and then we had just pulled on the triangle, and we suddenly realized that we had an equilateral triangle, where the bisector, which we had drawn before, was suddenly the perpendicular bisector. And you can do something like that with the program and explore such things, which is not possible on paper.

CASE 2: PETE ("INDEPENDENT POSITION")

[beliefs about MDC-predicative]: Pete stated that the world is constantly changing and progressing, and new and emerging DTs are part of this progress. He believed that it is totally fine, if students can only explain and think about mathematics with reference to DT. For him, the "*connected pos*ition" and the "*independent pos*ition" were actually quite the same – except that students may refer to different tools in their thinking or explanations:

Pete: For example, many students cannot describe angles without having a set square in their hands. And a set square is also a tool but not a digital one. And the digital tools, are just somehow a further development and they can be used gladly and can also be used gladly in justifications.

[beliefs about MDC-operational]: Pete said that students are currently required to do mathematics without DT in German schools by law, and therefore it is currently important that students can do mathematics without DT. If this was not the case, he explained, it would be perfectly fine for him, if a student could only do mathematics with DT. He argued that everything a student can do by hand can also be done with DT – e.g., solving equations. He stated that with DT, one can even do certain mathematical procedures in different ways. For example, plotting a function can be done directly by entering the expression or by creating a table of values and plotting these values, which he argued is very close to drawing a function by hand:

Pete: For example, a graphic calculator can also be used to insert tables or similar things and then a function can be drawn without just simply entering the function and it is there. So, they can also draw functions on the graphic calculator, like they would do it by hand, so to speak.

[beliefs about time point of DT use]: Pete strongly opposed the view that DT should only be used after the mathematics has been understood thoroughly without use of DT. This is because DT provides opportunities to learn, he said:

Pete: I think that DT offers a possibility for many students to access mathematics in a different way. And that gives them the opportunity, for example, to discover graphs or similar things that I [they] don't have the opportunity to do by hand. Then the process is faster. The process is maybe even clearer. I think there are many possibilities that help the students."

[beliefs about potentials of DT]: Pete strongly believed that DT supports the learning of mathematical concepts, and he refers to the points he made when answering the previous question. He augmented his position by referring to a personal experience during his own schooling, where his teacher had used DT for introducing the parameters of functions:

Pete: I actually noticed for myself that this [it] was a good access with the slider, because I saw exactly what was happening with the function, and it worked quickly. I understood relatively quickly and well what the individual parameters of the function were and what they were good for. This was a quick, understandable access for me.

DISCUSSION

In this case-study we have taken a first step to investigate how PSTs justify their beliefs about the relation between mathematical and digital competencies, and how their beliefs relate to other beliefs in their belief system. Lara's case shows that for PSTs "to understand mathematics" can mean to be able to think about, explain and do mathematics without DT, i.e., that students' predicative knowledge is not intertwined with DT. Consequently, a derivative belief of Lara is that a student should be able to explain and think about mathematics without any reference to DT ("*independent position*"). However, Lara is positive about using DT because of its interactivity and the potentials for discovery learning. Lara's belief that DT should only be used if the mathematics is thoroughly understood without DT, can be explained as a way to manage the tension of using DT (acting in line with her positive beliefs about DT), while at the same time maintaining an independence between mathematical understanding and DT (acting in line with her "*independent position*"). However, there seems to be some salient contradiction in her belief system, because a question arises as to what should be discovered by using DT, if concepts and relationships have already been taught without DT. Lara could also have considered the case of using DT to consolidate knowledge and deepen understanding of concepts following their introduction without DT.

In a preceding quantitative study, the authors of this paper found that many PSTs held a strong "*independent position*" (Thurm et al., 2022). This potentially becomes problematic, if the goal is that these PSTs support students to develop MDC. Lara's case indicates that teacher educators will need to discuss with students, what "mathematical understanding" means and how it can be witnessed. This will entail to make clear that if DT is used in an epistemic way (i.e., to create understanding or support learning within the user's cognitive system), the conceptual fields, and the set of situations that students use as points of reference to give meaning to a concept, would not be independent of DT. Also, PSTs need to understand that if students use DT extensively in epistemic ways (to understand mathematics) and pragmatic ways (to "do" mathematics), their operational knowledge will be inherently linked to DT.

We can conclude that it is not enough to focus on PSTs' beliefs about the potentials of DT and convince them of the potentials of DT. Rather Lara's case shows that PSTs' beliefs about the potentials of DT may be less central than other beliefs – leading to PSTs strongly opposing the notion of MDC based on fundamental beliefs about the nature of mathematical understanding in relation to DT, despite their positive beliefs about DT.

Pete's case illustrates that a "*connected position*" may be justified by placing DT in relation to other tools that have been used in mathematics education for a long time. For Pete "understanding mathematics" has nothing to do with which tool a student refers to in his or her explanations or when doing mathematics. Such a more "holistic" concept of mathematical understanding, i.e., one not bound to certain tools, could be beneficial for developing PSTs' beliefs about MDC towards a "*connected position*".

Tabach (2021) has pointed out that besides MDC for students the research community must try to provide a parallel conceptualization for teachers. Teachers' beliefs about MDC and beliefs about "mathematical understanding" in relation to DT indeed seem to be a crucial aspect of such a parallel conceptualization. Nonetheless, the present study is only a first step in investigating beliefs about MDC. We are currently analysing more cases to contribute to a better understanding of teachers' MDC related beliefs.

ACKNOWLEDGEMENT

Parts of this research have been supported with funding by UK Research and Innovation (UKRI) under the UK government's Horizon Europe funding guarantee for the Twinning Project 101078875: Transformative and Emerging Educational Technologies.

REFERENCES

Andrà, C., Liljedahl, P., Erens, R., & Rouleau, A. (2021). Tensions within Teachers' Beliefs: Implications for Teacher Professional Development. In Inprasitha, M., & Changsri, N., Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 9–16). PME.

Ferrari, A. (2012). *Digital competence in practice: An analysis of frameworks.* Joint Research Centre of the European Commission. https://data.europa.eu/doi/10.2791/82116

Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative inquiry, 12*(2), 219–245. https://doi.org/10.1177/1077800405284363

Geraniou, E., & Jankvist, U. T. (2019). Towards a definition of "mathematical digital competency". *Educational Studies in Mathematics*, *102*(1), 29–45. https://doi.org/10.1007/s10649-019-09893-8

Green, T. F. (1971). *The Activities of Teaching*. McGraw-Hill.

Guin, D., & Trouche, L. (1999). The Complex Process of Converting Tools into Mathematical Instruments: The Case of Calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195–227. https://doi.org/10.1023/A:1009892720043

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*(1), 9–28. https://doi.org/10.1007/s10649-019-09903-9

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Eds.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 257–315). IAP.

Rott, B. (2021). Inductive and deductive justification of knowledge: epistemological beliefs and critical thinking at the beginning of studying mathematics. *Educational Studies in Mathematics, 106*(1), 117–132. https://doi.org/10.1007/s10649-020-10004-1

Tabach, M. (2021). Competencies for teaching mathematics in the digital era: Are we ready to characterize them? In Inprasitha, M., Changsri, N., & Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 32–47). PME.

Thurm, D., & Barzel, B. (2020). Effects of a professional development program for teaching mathematics with technology on teachers' beliefs, self-efficacy and practices. *ZDM Mathematics Education, 52*(7), 1411–1422. https://doi.org/10.1007/s11858-020-01158-6

Thurm, D., & Barzel, B. (2022). Teaching mathematics with technology: a multidimensional analysis of teacher beliefs. *Educational Studies in Mathematics, 109*(1), 41–63. https://doi.org/10.1007/s10649-021-10072-x

Thurm, D., Geraniou, E., & Jankvist, U. T. (2022). Preservice teachers' beliefs about mathematical digital competency – a "hidden variable" in teaching mathematics with digital technology? In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti. (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12).* Free University of Bozen-Bolzano and ERME.

Vergnaud, G. (2009). The Theory of Conceptual Fields. *Human Development, 52*(2), 83–94. https://doi.org/10.1159/000202727