REPRESENTING DISTANCE-TIME SCENARIOS IN A DIGITAL EMBODIED LEARNING ENVIRONMENT

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The purpose of this study was to explore how students use the feedback provided by a digital embodied learning environment to graphically represent qualitative and quantitative aspects of distance-time scenarios and overcome typical graphing mistakes. Fifty-nine 11-year-old-students were asked to construct graphs for two scenarios and got feedback in form of an animation of the corresponding time-distance scenarios. The results indicate four different categories of how students used this feedback to effectively revise their work. Further, the analysis shows that students' interaction with the learning environment was beneficial since several students exhibited a covariation approach to distance-time scenarios when interpreting their graphs and grasped that the two quantities are varying simultaneously and every value of one quantity determines exactly one value of the other.

Keywords: covariation; functional thinking; embodied digital tools; graphical reasoning

INTRODUCTION

Researchers still struggle to find effective ways of integrating technology into mathematics teaching and learning (Polly, 2014). This also holds true for the area of functional thinking and graphical reasoning. It is a timely issue to examine how portable and handheld digital technologies offer opportunities for enacting embodied learning in functional thinking situations that require representing functional relations (Abrahamson & Bakker, 2016). Indeed, contemporary digital tools provide learning environments for understanding graphs of change and motion that incorporate students' own motion experiences. The present study focuses on students' competency of graph construction and to what extent motion experiences help students to evaluate the correctness of their graphs. The goal of this study, therefore, is to empirically investigate how students use the feedback of a digital and embodied learning environment to represent distance-time scenarios and overcome typical mistakes. To address this question, we explore how 11-year-old students revise their graphs that represent distance-time scenarios after watching the animation provided by the learning environment.

THEORETICAL BACKGROUND

Covariation and Graphical Reasoning

Distance-time scenarios mainly entail a covariation approach to functional thinking. The covariation approach is mostly studied by contextualized dynamic functional relationships (Thomson & Carlson, 2017). It is viewed as a way to make sense of relationships of dependence, causation, and correlation between quantities, and thinking in terms of rates of change. Creating the graph of a dynamic functional relationship requires reasoning about changes of dependent and independent variables, and the direction of these changes. Moreover, translations between natural language and graphical representations to model dynamic functional situations and imaging and coordinating the simultaneous changes of the involved values are crucial (Ellis, 2011).

The covariational reasoning framework, proposed by Thompson and Carlson (2017), suggests that students should be able to think about how two quantities vary and to understand that the two quantities vary simultaneously. Indeed, students' ability to reason with quantities and relationships fosters their functional thinking (Moore et al., 2013; Pittalis et al., 2020). Covariational reasoning involves the mental coordination of the values of two quantities. Covariational reasoning can contribute towards developing graphical reasoning, since it entails assimilating a graph as a trace in progress (Moore et al., 2019). However, the interpretation of graphical representations that describe dynamic scenarios can be challenging for students (Friel et al., 2001). Discerning among discrete and continuous representations of change and differentiating between the shape of a graph and the characteristics of the scenario pose various difficulties.

Graphical reasoning encompasses both graph interpretation and construction. Graph interpretation consists of three essential processes: recognizing visual features of a graph, interpreting relationships represented by these features and connecting the identified relationships with what the graph represents (Shah & Hoeffner, 2002). For distance-time scenarios, students should be given the opportunity to connect the physical situation (i.e., own motion experiences) with visual elements of the graphical representation and vice versa. To do so, two instructional approaches have been suggested. The first one emphasizes on quantitative or local aspects of graphing, while the second one underlies the importance of grasping qualitative or global aspects. The latter one highlights the graph's general shape as it facilitates visualizing the relationship between the quantities before constructing a graph, as shapes of trends are mapped onto the graphs' axes. It relates to Castillo-Garsow et al.'s (2013) description of thinking about the relationship between two variables as continuously changing. This type of thinking models the notion of motion and facilitates mapping young students' everyday experiences with motion to the abstract concept of continuous change. Research showed that students encounter difficulties when graphically represent changes over time, e.g., when interpreting a graph as a representation of a real event or when plotting points of a graph without considering the values in between and consequently without understanding that the graph represents a relationship between continuous covarying quantities (Thompson & Carlson, 2017).

Embodied Learning Environments for Graphing Motion

A systematic review by Duijzer et al., (2019) shows that embodied learning environments which immediately link students' own motion with the mathematical representation provided by a digital tool are effective in supporting students' understanding of graphs. To become familiar with the graphical representation of scenarios involving covarying quantities, it would also be possible to use dynamic simulations. Therein, students can either focus on single points to connect different types of representations or manipulate variables and observe the consequences. In the latter possibility, students can observe the effect of a systematic variation in all representations simultaneously, which make the covariation of quantities perceptible. A digital environment that links a graph with an animated motion in real-time provides a valuable entry-point into reasoning about continuous change represented in the distance-time graph, because the motion observed corresponds to a time value at every point in time. For the case of graphing motion, digital learning environments can be categorized in respect to bodily involvement and immediacy (Duijzer, et al. 2019). Bodily involvement can be distinguished between own motion and observing others/objects' motion. Immediacy is defined in terms of immediate and non-immediate. An immediate task provides a simultaneous interaction with the physical environment while the graph is plotted, whereas in the second case this interaction is based on an embodied simulation. Research showed that embodied learning environments, which make use of students' own motion immediately linked to its graphical representation were more effective (Duijzer, et al. 2019). In the present study, we turn attention on students’ understanding of graphing motion as they observed and influenced an objects’ motion in which the bodily involvement (of the animated object) took place in the absence of direct environmental stimuli.

THE PRESENT STUDY

The purpose of this study was to investigate how students use the feedback provided by a digital embodied learning environment to represent qualitative and quantitative aspects of co-varying quantities (distance-time scenarios) and overcome typical graphing mistakes. The research questions are: (a) How do students use the feedback provided by the digital learning environment to revise their graphs and (b) to what extent does students’ interaction with the digital environment facilitate the emergence of a covariation approach to distance-time scenarios?

Participants, Intervention, Evaluation and Procedure

The participants of this study were fifty-nine 11-year-old-students, 28 girls and 31 boys from three Grade 5 classrooms. Consent to participate in this study was given by the students and their parents. They exhibited a broad spectrum of academic achievement levels. Students had used tablets in mathematics several times before, but they had not previously been taught functional relationships involving measures that covariate simultaneously.

As intervention context, the research team designed a module with an emphasis on the covariation aspect of functional thinking consisting of four 40-minute lessons. The lessons were delivered by one of the members of the team in a two-week period. They were designed based on four principles: situatedness, inquiry-based learning, embodiment, and utilization of digital tools. The module included embodied activities that required conceiving co-varying quantities, representing graphically distance-time scenarios (with paper and pencil and digitally), and animating with bodily movement distance-time graphs that were developed in the framework of the FunThink project. While working on the module, all students were given an activity worksheet and tablets with the online applet *Turtle Crossing* from the Desmos platform (desmos.com). The applet presents a turtle that walks away from the sea and students had to make connections between several turtle-crossing scenarios and the respective distance-time graphs. In the present study we used the functionality of the applet to draw a distance-time graph and then watch an animation of the turtle’s corresponding journey (see Figure 1 right). The user can pause the animation, scroll the play-slider back and forward, and observe in the form of a vertical line the time-correspondence of the animation with the graph. When the user draws the graph, a sign displaying the position of the turtle in the animation-setting appears.

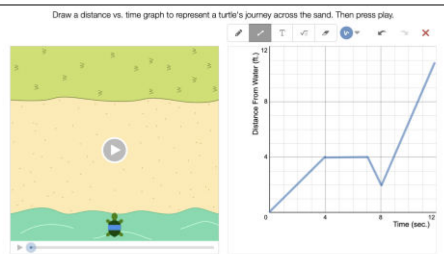
Turtle-crossing scenarios	Desmos environment
Task 1 focuses on global aspects of the graph: The turtle moves away from the sea. Suddenly, it stops for a while. Then, it continues moving away from the sea. Task 2 focuses on local aspects of the graph: The turtle moves 8 ft out of the sea in 4 seconds. It pauses for 2 seconds. It then returns to the sea in just 2 s.	

Table 1: Description of Tasks in the digital Learning Environment

Two weeks after the completion of the intervention program, each student was interviewed in a session that lasted approximately 20 minutes to evaluate students’ learning. Students were asked to construct four graphs for given scenarios in the *Turtle Crossing* applet. They were informed that they could revise their work based on the feedback provided by the animation. Two of the scenarios required conceptualizing global aspects of the graph while the other two involved grasping local aspects. In the present study we analyze students' work in two of these tasks (see Table 1 left).

Data Collection and Analysis

This study used qualitative methods for data collection. We videotaped students' work on the tablets to capture their actions and oral explanations while working. A qualitative interpretive framework was used in the analysis of the data (Miles & Huberman, 1994). We used the constant comparative method to compare students' work and formulate categories of students' ways to use the provided feedback and explanations. The categorization was developed within an iterative back and forth process, that was both literature-based as well as data driven.

RESULTS

We addressed the first research question of the study by examining how students revise their graphs based on the feedback provided by the digital learning environment. We examined in depth the sequence of actions of the students that did not construct a correct graph based on the given scenario from their first attempt. We present here the categorization of students' ways of utilizing the provided feedback based on our analysis: (a) Delete the whole graph and start over after watching the animated turtle trip (without identifying which are the correct and the wrong parts of the graph), (b) delete and revise the whole graph or parts of the graph to correct the wrong parts of the graph, by focusing mainly on the animated turtle trip (keeping or reconstructing the right parts of the graph), (c) delete only the perceived wrong part of the graph and revise it by interpreting the effect of this part of the graph on the animated turtle trip, and (d) validate the graph section by section and revise selected parts by controlling the enactment of the route thought the use of the play/pause button of the animation. Below, we illustrate the four categories with students' answers.

In Task 1, 32 out of the 59 students revised their work (the remaining students provided a correct graph in their first attempt). Twelve of them correctly revised their graph after one or two corrections, fifteen after multiple attempts and the rest of them did not manage to provide a correct answer. The students that provided a correct graph after multiple attempts or did not manage to do so could mainly be assigned to the first two categories, while the revising procedure took a long time. This may have resulted from the fact that students revised their work based solely on evaluating whether the animated trip of the turtle corresponds to the given scenario, without making direct links between each part of the graph with the animated trip and the scenario. Figure 1 presents Helen's attempts to construct the graph in Task 1. Helen's work is indicative of students that used the first two categories in their attempt to revise their graphs. Their work suggests an iconic perspective of graphs or difficulty to coordinate the different directions of change of the two quantities. First, Helen constructed the graph shown in Figure 1i(a). Then, she selected the play button to see the animated turtle journey. When she was asked to comment on the animation, she responded that the turtle disappears instead of staying still. She explained that the disappearance of the turtle was due to the fact there was a blank space between the two parts of the graph. Afterwards, she selected the "delete all" button of the app and started over to create a new graph. She constructed the graph presented in Figure 1i(b). While watching the animated turtle movement, she did not figure out the new trip immediately, so she decided to watch the animated video two more times. Then, she commented that the turtle moves forward faster than staying still, without trying to match the different parts of the graph to the turtle's move. She started over again and constructed a graph representing the stop interval as a vertical line, explaining that leaving blank space does not work as the turtle disappears or moves fast (Figure 1i(c)). Again, she watched three times the animated video, without noticing the difference of this video to the previous one and commented that the turtle flashes out and then run fast again. This time, she explained that the graph should stop and start again and constructed the graph presented in Figure 1i(d). After watching the new animation twice, she concluded that she should have represented the blank space in the graph in a different way because her construction made the stop invisible. Then, she made a different graph, by representing the stop interval with a dotted line parallel to the horizontal axis. When she checked the new graph, she explained that it matches the scenario as the turtle stays still for a while. When the interviewer prompted her to rethink why the turtle was flashing on and off during the stop, she answered that the flashing of the turtle intended to emphasize its stop.

Figure 1 also presents the graphs made by Stella in Task 1. Stella's work is indicative of students that deleted only the perceived wrong part of the graph and revised it by interpreting the effect of the wrong part of the graph on the animated turtle trip (third category). First, Stella constructed two diagonal segments to represent the scenario "the turtle moves forward" and left a blank space for the turtles' stop (see Figure 1ii(a)). After watching the animated video twice, she explained that the turtle disappeared instead of making a stop for some seconds. When asked to explain the gap in her graph, she answered that the turtle should stay in the same position for a while, thus, she decided to "continue the line after a while from the same position" (indicating the corresponding y-value). Then, she used the segment tool and constructed a dotted horizontal line to represent the turtle's stop (Figure 1ii(b)). Watching the animated video helped her conceptualize that the dotted line resulted in the appearance and disappearance of the turtle. Then, she used the "eraser" tool to delete the dotted line and sketched a horizontal segment instead, explaining that "I used previously a dotted line for the stop, but it did not work, when the line of the graph stops the turtle disappears, I must make a segment like this (indicating a horizontal direction) to connect the two parts that the turtle moves forward".

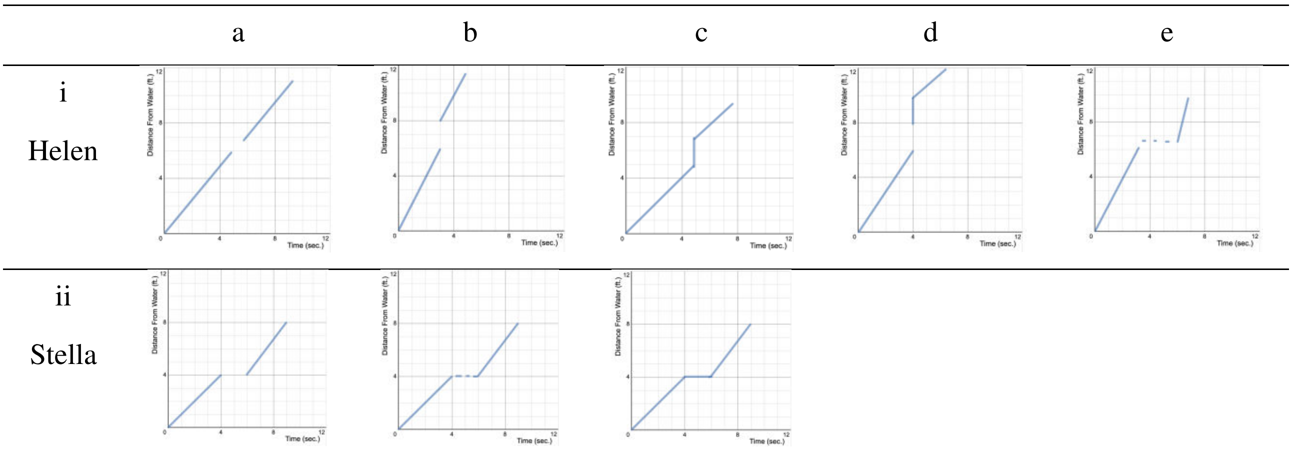


Figure 1: Helen and Stella's graphs in Task 1

In Task 2, 35 out of the 59 students revised their work. Twelve of them correctly revised their graph after one or two corrections, seventeen after multiple attempts and the rest of them did not manage to provide a correct answer. Figure 2 presents Andrew's graphs in Task 2. Andrew's work is indicative of the second category, as he revised only the part of the graph that did not correspond to the scenario. First, Andrew constructed the graph presented in Figure 2i(a), as he represented the scenario "returns to the sea" by extending the line graph to the origin. After the first attempt, he watched the animated video three times because he was surprised by the fact that the animation included two turtles and thought that there was a bug in the app. He explained that he could not figure out the problem and the best option was to delete a part of the graph. Then, he deleted a segment of the graph and watched the animation of the new graph (Figure 2i(b)). He explained that he realized what was going wrong and that one part of the graph (Figure 2i(b)) should be kept, as it represents the first two parts of the scenario and add a third segment to show the return of the turtle to the sea. However, he did not interpret the effect of the segment he deleted to the route of the turtle and the reason of the appearance of two turtles in the first graph. Consequently, he used a trial-and-error procedure to add a third segment to the graph (see Figures 2i(c)-(e)). After watching the animated video of the last graph, he explained that his graph was correct, besides the fact that the duration of the return of the turtle was six instead of two seconds. It seems that as the graph with a vertical segment did not work, he opted to construct a diagonal segment till the last value of the axis, emphasizing the direction of the segment and ignoring the numerical constraints (global view of the graph).

Zoi's work is indicative of the fourth category as she efficiently used the play/pause button of the animation to revise her graph. In Zoi's first attempt, she seemed to have translated the statement "the turtle returns to the sea in just 2 s" as one that reaches the sea in the 2 seconds (see Figure 2ii(a)-(b)). Zoi watched the animated video many times and paused the video when a second turtle appeared. When asked to explain why two turtles appear from the second to the sixth second, she scrolled the slider of the play button back and forward and observed the vertical line in the graph that shows the time-correspondence of the video and the graph. She observed that during this time-interval (the appearance of two turtles) the vertical line crosses the graph twice. Then, she deleted the wrong segment, played the video, and paused at the 6th second. Afterwards, she started constructing a diagonal segment to represent the return of the turtle and watched at the same time the dotted line in the left screen that displays the position of the turtle along the time (see Figure 2ii(c)-(d)). Based on the feedback, she completed the graph. She played again the video and used the pause button at the fourth, sixth and eighth second to validate her graph in terms of time-accuracy.

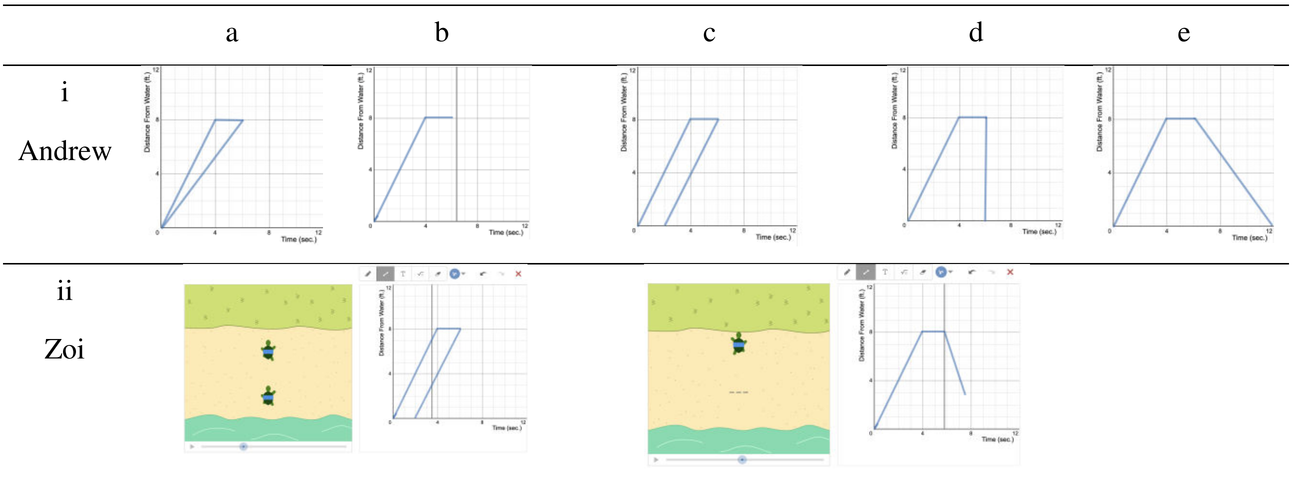


Figure 2: Andrew and Zoi's graphs in Task 2

The second research question examined to what extent does students' interaction with the digital environment facilitate the emergence of a covariation approach to distance-time scenarios. The analysis showed that specific features of the app facilitated conceptualizing that (a) the two quantities (distance and time) vary simultaneously, (b) every value of one quantity determines exactly one value of the other, and (c) the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant. Below, we present examples of such instances. In Task 1, several students represented the situation “the turtle stops for a while” by leaving a blank space or making a dotted line. While watching the animated trip of the turtle, students observed that leaving a blank space or a dotted line results in the disappearance of the turtle from the screen. Students commented that “I thought that if there is a space between them [meaning the two other parts of the graph], the turtle would stop, but it disappeared, that’s not possible, we cannot disappear the turtle, we are not magicians, the turtle should appear from the beginning to the end”, “I killed the turtle, poor turtle, the turtle should keep going or just stand watching all the time”, and “what happened to turtle?... I left a blank space for two seconds to show that it rests for two seconds, but it did not work, I wanted to show that the distance is the same for two seconds”. We conclude that these three examples indicate an intuitive understanding of the fact that distance and time should vary simultaneously in this scenario, as students mention that the turtle should appear throughout the video. In Task 2, the most common mistake was to represent the return of the turtle by constructing a segment from right to left (see graphs in Figure 2ii). In that case, two turtles appear in the animated video. Students shared the following interpretations: “Oh no… two turtles, when did it give birth? … it seems that the app gets two positions of the turtle in the same second and gets confused”, “I played the video back and forward, I made this [showing the back segment] to show that the turtle returns, but two turtles appeared from the beginning, the second turtle shows the return…wrong timing”, “it is wrong, I managed to return time back! Look [displaying a vertical line for the third second], in the third second the turtle had to be in two places simultaneously, so it copied itself”. These interpretations indicate an emergent understanding of the fact that every value of one quantity determines exactly one value of the other quantity. In terms of the specific scenario, in each point in time the turtle can appear in only one distance point away from the sea. A variety of explanations emphasized that time runs, while distance increases when the turtle moves forward and decreases when it returns to the sea. These explanations provided evidence of students’ interpretations using their own wording that the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant.

DISCUSSION

The contribution of this study lies on the empirical examination of the way students use the feedback provided by a digital embodied learning environment to represent qualitative and quantitative aspects of distance-time scenarios. The analysis showed four different categories in respect to the extent to which students used the provided feedback, in the form of an animated video of the constructed graph, to effectively revise and interpret their work. Revising based on the first way included deleting the whole graph and starting over after watching the animated trip. The difference of the second way compared to the first one was editing only the wrong parts of their graph. The third way was different in a qualitative way, as students revised the wrong part of the graph by interpreting the effect of this on the animated trip. Finally, the fourth way took full advantage of the environment functionalities, such as utilizing the play/pause button to validate the graph section by section. Further, the analysis showed that students’ interaction with the learning environment was beneficial since several students exhibited a covariation approach to distance-time scenarios. They connected the animated video with the constructed graph. Their interpretation showed an understanding that (a) the two quantities (distance and time) are varying simultaneously, (b) every value of one quantity determines exactly one value of the other and (c) the quantity of time is constantly increasing, while the quantity of distance may increase, decrease, or stay constant (Thompson & Carlson, 2017).

These results could be valuable for educators, curriculum, and software developers as the study showed the potential of embodied digital learning environments to connect graphs with a corresponding real-life animation. It seems that the animated video contributes to an emergent intuitive understanding of difficult mathematical concepts, such as the covariation approach to functional relationships. For instance, observing that a single turtle cannot be in two different positions at the same time helped visualizing that every value of one quantity determines exactly one value of the other. Further, the study highlights the need of adopting appropriate didactical approaches and the role of the teacher to maximize the learning benefit for students (Polly, 2014). Several students did not use all the functionalities of the learning environment, validated superficially their work based on the animated video, and did not attempt to develop an in-depth understanding of the effect that each part of the graph has on the turtle's trip. Targeted guidance and appropriate questions would prompt students to link the available representations and reason about the effect of the form and the direction of each part of a graph in a real-life scenario and grasp the corresponding limitations in terms of the involved varying-quantities. This teaching approach could exemplify the importance that learning environments hold as means to concretize important mathematical concepts.

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