EXPLORING THE POTENTIAL OF 3D PRINTING AT MATHEMATICAL LITTERACY

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Three-dimensional (3D) printing refers to a technological procedure that turns computer-based digital files into solid objects. This study attempts to explore to what extend 3D printing could be used as a learning and educational tool, assisting students to understand mathematics and raise their environmental awareness. We claim that the use of 3D printing in education will revolutionize the methods and concepts related to education and will assist students to gain a better understanding of scientific and experimental concepts. In this paper, we present and discuss our findings from a two year project, run at three primary schools in Greece, Italy and Portugal. A total number of 512 students and 34 teachers participated in this experiment and as indicated by the research results 3D printing could modify mathematical literacy and improve the students' capabilities and skills.

Keywords: 3D Printing, Mathematics, Recycle, Educational Curricula

INTRODUCTION

Three dimensional (3D) printing is a process that turns digital files into solid objects using a desktop 3D printer. Once the digital files are created, either by using computer design software or a 3D scanner, they are sliced into sections and a 3D printer stacks the raw material into layers.

3D printing is a technology that enables hobbyists to turn imagination into reality. According to Lipson and Kurman (2012), this technology will close the gap that divides the virtual and the physical worlds. 3D printing offers the promise of gaining control over the physical world because it provides powerful new design and production tools.

Alpern (2010) notes that "… all you have to do is to load a file and you can replicate shapes that are not manufacturable through traditional methods. All this is called a 'flexible factory in the box'. 3D printing has a wide range of applications in medicine, education, culture, the clothing and footwear industry, as well as in the army. Items, such as body organs, upper and lower limbs, dentures, guns, toys, shoes, jewelry, and statues can be produced using additive manufacturing (Martorelli et al., 2013; Rankin et al., 2014).

This article describes the findings of a project, funded by the European Commission, whose duration was thirty months (November 2019 to April 2022). The main goal of the project was to support students to integrate mathematics into their daily lives, to overcome "fear" of applying mathematical concepts & operations and finally to understand relevant theories in more favorable terms. The Project aimed to enhance the imagination and the creativity of students and to strengthen their skills. Subsequently, they became familiar with the concepts of recycling and reuse of materials. The knowledge formed in the fields of mathematics, 3D printing & recycling, is transformed into educational material, which was used, in the 2nd phase, in order to address the identified needs.

With reference to the structure of this paper: Section 2 presents a literature review of studies focusing on 3D printing in education, and a brief overview of Mathematical Learning Difficulties;

Section 3 formulates our research questions and the methodology that was used; Section 4 presents the results of the implementation of the educational scenarios at the schools of the three participating countries and Section 5 summarizes the results and suggests areas for further research.

LITTERATURE REVIEW

3D printing uses a new approach in education. Recently, schools and universities have provided students and faculty with new labs that are equipped with 3D printers and 3D scanners, while new educational frameworks have been developed that incorporate this new manufacturing technology into learning and teaching activities. Until recently, few students had access to this new technology due to cost related limitations. As 3D printers and scanners become more affordable, universities and schools are in a better position to purchase more digital manufacturing systems and to train their students in using this technology (Junk et al., 2015).

In the European Union, the mathematics curriculum for primary, secondary, and tertiary education aims to help students apply their mathematical knowledge and skills to solve problems and make decisions. Students usually learn more by "writing" mathematics and less by "thinking" as mathematician. This means that it is common for students to learn difficult techniques in order to solve equations, but they do not really understand how to utilize these concepts to solve real-life problems. Thus, there is a gap between the goals of mathematical curricula and the learning outcomes.

Many students face difficulties in mathematics. According to Shalev et al., (2000), this represents 3–6% of all children. Students in this group have comprehension problems and face challenges trying to solve arithmetic problems or problems associated with disordered numbering. This group of students is heterogeneous, and it encompasses many subgroups, each of which is characterized by a different learning disability (Bartelet, 2014). Rousselle et al. (2007) claim that students with MDL exhibit difficulties in connecting symbolic representations of numerical magnitude, such as Arabic numerals, with the non-symbolic quantities that they represent. Some studies have found a significant correlation between MLD and verbal working memory deficits in children (Roselli et al., 2010). Some children also find it difficult to do comparison tasks (Mundy et al, 2009). The numerical distance effect, determining the distance between closer numbers in comparison to the distance between numbers that are far apart, refers to the difficulty of determining the numeric properties of a set of items in the real world (Fayol et al., 2005). According to Prathana, et al. (2014), the lack of ability to read is the biggest obstacle to students' comprehension. If students cannot understand the meaning of the text, they cannot proceed to solve the problem.

To summarize, students face a number of difficulties in learning mathematics due to: 1) the course syllabus, which is sometimes advanced, 2) the teaching strategy that teachers use in the classroom, 3) the difficulty of the assignments (Kavkler, 2014), or 4) the traditional way of teaching (Cabrita et al., 2015) in which students learn how to solve math equations without teaching them ways to utilize them in everyday life.

It is our belief that 3D printing could help students through engineering projects to convert a problem of building a house to a physical prototyping, something which aims them to understand mathematical concepts like straight and parallel lines, measurements, distance from a specific point. By printing in three dimensions' students with arithmetic disabilities could sketch and print for example the Tower of Anoi and understand the different numbers and different scheme sizes. More specifically, teaching through the use of 3D printing enables teachers to shift the students' interest and caution from mathematics to technology and better understand mathematical concepts.

RESEARCH QUESTIONS & METHODOLOGY

The current paper discusses the results of the 3D-ReMath project, whose goal was to assist pupils to better comprehend mathematical concepts and empower mathematical thinking, by using the innovative production technology of 3D printing. Another area of focus, apart from technology, was the protection of the environment through recycling. Moreover, the necessary material for 3D printing, called PLA (plastic), could be replaced by a special thread that would be created by students with recycled material, thus covering both needs. In this light, the application for the 3D-ReMath Project was based on the assumption that 3D printing could assist primary and secondary school students better understand mathematics and at the same time become familiar with the concept of recycling.

The contribution of 3D-ReMath was based on pupils' participation in recycling initiatives. By collecting plastic that was later reused, students had the chance to explore a new world of solid objects and through the use of technology they managed to increase their learning ability in maths. Difficulties in mathematics produce discrimination among students. It is widely accepted that technology and its new applications motivates pupils.

3D-ReMath was totally aligned with STEM's (Science, Technology Engineering & Maths) philosophy (Do questions like Scientist, Design like a technology, Print (Build) something solid like an engineer, Deduce like a mathematician). The innovation of the program therefore lies in the fact that it managed to combine three different concepts, to create corresponding educational material and to train teachers and students with multiple and beneficial fields such as: 3D printing (cost reduction by creating a special thread instead of buying it), mathematics (introduction of an educational method to reduce inequalities) and ecology (raising awareness on reuse of materials and recycling).

In order to meet the set goals, several steps preceded: (a) the development of a Map for math disabilities, (b) the development of curricula for Mathematics, 3D Printing and Recycling and (c) two rounds of pupils' activities in the three distinct modules.

A map identifying math difficulties was developed based on the teachers' experience and on a sample of students from all schools. In order to facilitate communication and discussion of ideas between partners, an online forum was created. Through this tool, each partner proposed several questions for teachers' questionnaires for describing mathematics difficulties. After the different proposals were analyzed and debated, ten questions were selected for the teachers' questionnaires. Two questionnaires were developed: one for teachers who taught students aged 6 to 10 and one for teachers who taught students aged from 11 to 14. Online surveys were created on google forms and collected responses were imported into SPSS for data analysis. After analyzing the teachers' questionnaires and identifying maths difficulties from their perspective, these were grouped into categories. Then, each partner proposed several multiple choice questions for the students' questionnaires, based on the teachers' questionnaires, and grouped in categories, which led to ten questions for each questionnaire. Based on the students' population of the three schools, the size of the sample per gender and school was defined. The students' questionnaires were created on google forms and applied to this sample. Finally, based on the results of this study, a map for math difficulties was created in CMAP Tools. The conceptual map contained 10 examples of questions related to each identified difficulty. All partners actively participated with quality contributions for the elaboration of the difficulties map. Figure 1 presents the produced map.

Based on the difficulties, which were defined at the map, we proceeded to the development of three Curricula for mathematics' module. Each curriculum examined a different mathematical topic. The addressed issues were: (a) Fractions, (b) Mathematical Thinking and (c) Stereometry. The structure of curricula involved the description of the relation between a real problem and a maths problem and Learning Objectives of each lecture. Websites, e-books, open source software, books, videos, tutorials were also employed in order to support these curricula. A series of activities and exercises for increasing learning ability were employed and practical exercises were solved by using open source software like Geogebra and corresponding 3D printing activities at the Laboratory.

Through this process it became clear that students were motivated to learn how to use a 3D printer and better understood mathematics. Students were trained at the use of 3D printers and were acquainted with the 3D printing process, at designing 3D models using Tinker CAD and Sketch Fab. For the purpose of this project, XYZ DaVinci w+ 3D Printers were used and the method of Fused Filament Fabrication. This method turns digital files into solid objects by using thermoplastic material. Digital files are created, either by using computer design software or a 3D scanner, and once these files are uploaded to the 3D printer's software, they are sliced into sections and the 3D printer produces the object by stacking the raw material into layers. Throughout the pupils' training activities, we used PLA filament (plastic).

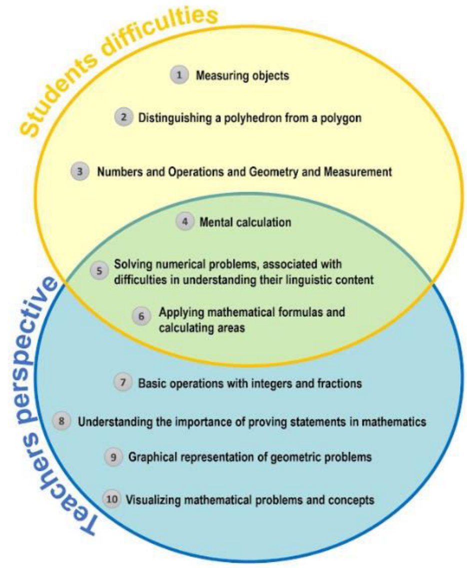


Figure 1: Map of mathematical difficulties

In order to raise the students' awareness concerning recycling, we urged them to collect plastic water bottles and transform the plastic material to filament that could be used for printing objects for mathematics' module i.e pies, numbers, letters, etc. Pupils were introduced to recyclable materials, such as plastic, paper and glass, and recognized the importance of waste management. Figure 2 presents the procedure of transforming plastic bottles to filament.

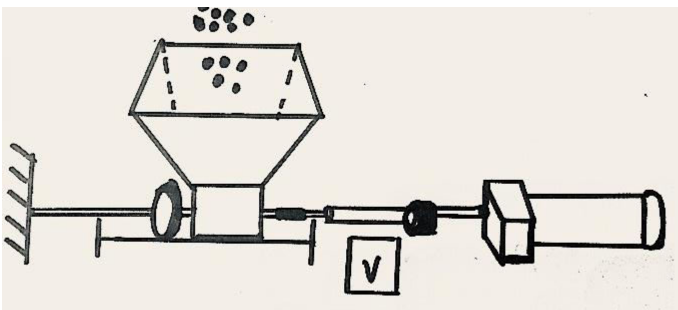


Figure 2: Mechanism of 3D filament

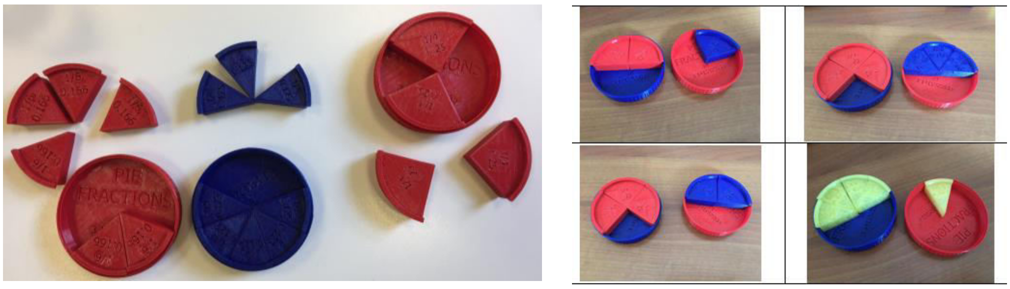
Using the above mechanism, after the plastic is collected and cleaned thoroughly, it is cut into small pieces, transforming it into a filament suitable for a 3D printer. The shavings are then placed in the hopper and are melted at a very high temperature. The produced mixture then starts to freeze and is directed through a tube of small cross-section into the rotating tube and wrapped around it in order to proceed to the filament.

RESULTS

The students who faced difficulties in mathematics and participated in the project joined a specific educational pilot program in order to improve their understanding of mathematical concepts and eventually integrate these concepts in their daily lives, assisting them in solving everyday problems. To achieve this goal, they were trained in the use of 3D printing and became familiar with the concept of recycling and other ecological aspects.

As mentioned above, three different mathematical topics were examined: fractions, combinations, and stereometry. Concerning fractions, the learning outcomes were: (a) Compare and order fractions and mixed numbers (b) Represent equivalent fractions (c) Identify the division statement that represents a fraction (d) Apply techniques of converting fractions into other, equivalent ones, (e) Simplify the numerator and denominator of a fraction by dividing them with the same divisor and (f) Add and subtract fractions having like and unlike denominators.

Through the use of a 3D printer, students printed a pan of pizza, of four, six and eight parts (pieces) as depicted on Figure 3 (a).



(a) (b)

Figure 3: Printed Pizzas and compare fractions

They compared fractions like 2/4 and 3/4, or 1/8 and 2/4 and also used the pizza pieces to perform calculations; i.e. addition or subtraction of fractions (Figure 4).

$\frac{1}{6}+\frac{3}{6}$ $\frac{1}{4}+\frac{3}{4}$ $\frac{4}{8}-\frac{1}{8}$

Figure 4: Calculations

Students were also taught about sets, subsets and combinations by printing numbers and letters. They also learned the concept of grouping objects based on a criterion e.g. color, name, etc. In terms of mathematical achievement, students learned to (a) understand the concept of Sets and Combinations (b) distinguish objects based on a criterion and (c) identify differences between sets and combinations of objects.

Set of numbers | Group of solid numbers in sets according to color | Group of printed numbers based on number

Figure 5: Sets and Groups

Concerning the combinations, students printed cubes in different colors, red, green, black and white. They used these four cubes and the noted down the ways in which they could be used, when repetition was allowed.

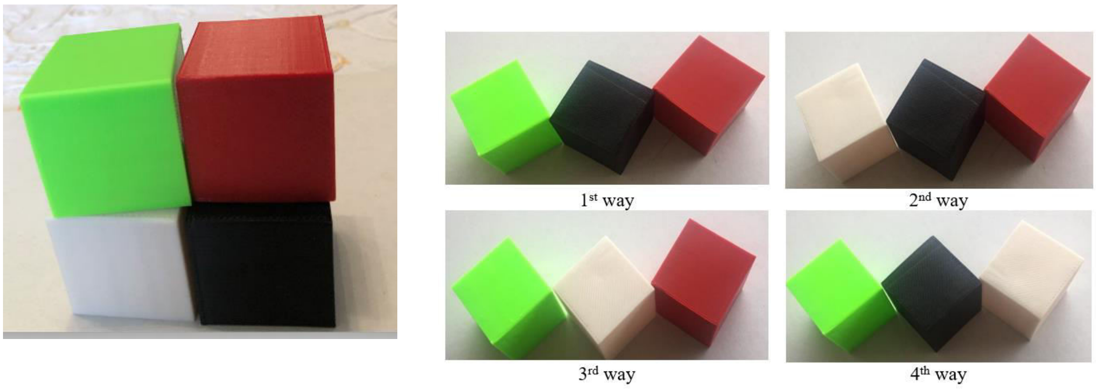


Figure 6: Combinations

Finally, the Learning Outcomes of Stereometry's curricula were to (a) develop spatial visualization, (b) distinguish among different criteria, (c) realize and apply mathematical formulas (d) Perform simple calculations. In terms of mathematical achievement, at the end of the training activities, students were able to (1) understand the difference between 2 and 3 dimensions (2) distinguish 2D and 3D shapes (3) calculate perimeter and area of 2D shapes (4) calculate surface area and volume of 3D shapes. At the Lab, students printed the followings shapes and then attempted to match the 2d shape with the corresponding 3d shape. (Figure 7).

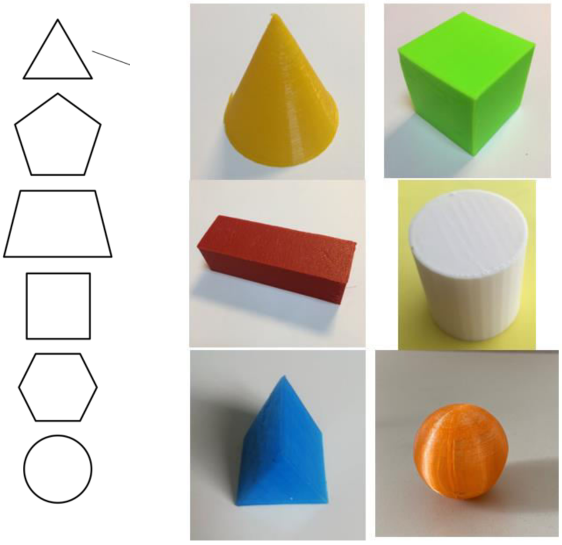


Figure 7:3D shapes

EVALUATION- DISCUSSION

Both the project and the proposed activities were evaluated through the use of questionnaires regarding: a. the above Curricula and b. their effectiveness in addressing the identified educational needs.

The students' questionnaire contained 10 questions, from which 8 questions referred to their learning experience, as well as the relative level of their satisfaction, while the last 2 questions concerned demographic issues. Out of the total number of 512 students that participated in the activities, we choose randomly 197 students, through the use of stratified sampling method. The two used data were "age" and "mother-country". The questionnaire was filled by 197 students, half of which were male. Concerning the involvement of each country in the project, 17% of the students came from Greece, 49% from Italy and 34% from Portugal. The collected data was analyzed through the use of IBM SPSS software. In this section we present the results concerning the pupils' satisfaction from their participation in the program, their belief on the contribution of 3D printing to mathematical understanding and their perception about the usefulness of the training program.

Based on the analysis, 88% of participating students enjoyed strongly their involvement in the program. For Greece and Italy, the level of satisfaction was rated from "enough" to "very much". Specifically, from the students that responded that they enjoyed the training activities very much 14% came from Greece, 45% came from Italy and the remaining percentage from Portugal.

Students were asked if they believed 3D printing helped them to better understand Mathematics. We used a four -scale Likert answer ("Not at all", "Not much", "Enough", and "Very Much"). Out of the 197? students 43% answered "very much", 40% answered "enough" (30%), 14% answered "not much" and 3% answered "not at all". The majority of Portuguese students (83%) gave positive feedback about the use of 3D printing technology in overcoming difficulties in maths. Additionally, 71% of Greek students believe that 3D printing helped them a lot (Enough"/ "Very Much") to understand Mathematics, while 35% of Italian students answered that 3D printing helped them "very much", 43% answered "enough" and the remaining percentage of 22% gave negative feedback.

Finally, the perception of pupils on the usefulness of the training course is overall positive as 74% of students answered "very much" and 23% "enough". Only 3 negative responses ("not at all") were recorded out of the 197 students, two of which came from Greek students and one from a Portuguese student. The corresponding question was: "Do you think that the topics covered and the teachings you have received in these courses will be useful for your future?" and a Likert scale of 3 items was used "Not at all", "Enough" and "Very much".

Regarding the distribution per country, there replies of the three countries seem to be equally divided with most students answering that the educational topics were very useful for future reference and use. Specifically, 57% of the students from Greece found the project's content very useful for the future, while 13% of them didn't find it adequate enough. The other students considered the program useful enough for the future. Additionally, 72% of Italian students claimed that the subject was very useful, while 2 out of 10 stated that it was adequately useful for the future and only 8% students did not consider the training program enough for further future use. Concerning the Portuguese team; 38% of students found the training program useful enough, while 57% believe that they will refer in future to the addressed topics of the program.

Based on the overall evaluation of the actions in which students participated, it appears that they helped them to achieve the purpose of supporting them in mathematics while becoming acquainted with the world of recycling and 3D printing.

CONCLUSION

In this paper, we briefly present a pilot training program based on the axes of education and the use of modern technology with a dominant feature of innovation. In particular, we introduced an alternative educational methodological model aimed at eliminating difficulties in mathematics. A method that utilized innovative manufacturing technology (3D printing), in order for students to understand mathematics, to increase their learning ability and at the same time their knowledge about recycling. Students exploring technology were able to improve their learning ability in mathematics while their awareness was raised regarding the protection of the environment through the reuse of materials and recycling.

One potential area for future research could be that of mathematical difficulties of students with special needs. One aspect that cannot be overlooked is the limitations associated with the challenges of familiarizing teachers with 3D printing technology and the high cost of the equipment that universities and schools incur when purchasing 3D printers and required materials (filaments). Setting aside these limitations, the paper tries to shed light on how traditional educational approaches will be transformed by technology and especially by this innovative technology of 3D printing technology.

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