PROGRAMMING AND COMPUTATIONAL THINKING IN MATHEMATICAL SUBJECT AREAS

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In England, computing is a subject with no formal relation to mathematics. However, several national projects have explored such a possible relationship and shared concrete materials, which has contributed to a general awareness of the potential for programming and computational thinking (PCT) in math. In this paper, we study whether and how English teachers include PCT as part of their mathematics teaching in a context where there are no legislative curricular requirements to do so. The study finds that English teachers mostly teach PCT as analogue computational thinking and that they are more likely to do so in the mathematical subject areas of functions, statistics, and algebra. We argue that the high frequency of statistics and the low frequency of geometry are interesting as they conflict with what research has found and discuss whether the English prevalence of analogue teaching of PCT can explain this.

Keywords: Subject Areas, Programming, Computational thinking

INTRODUCTION

Over the last 50 years, the idea that programming and computational thinking (PCT) hold the potential for students and practitioners to engage with mathematics in new ways has been expressed in various studies (e.g., Benton et al., 2018; Papert, 1982). More recently, PCT has also gained momentum in curricular revisions, where it has become part of the mathematics curriculum in countries such as France and Sweden (Bocconi et al., 2022). There are notable differences between countries that include PCT in their curricular revisions, in terms of what PCT content is emphasized and with what mathematical content it is coupled. As found in the study by Tamborg (2022), these decisions are not necessarily the product of carefully thought-out considerations but rather choices that are influenced to some extent by contingency in the policy-making process. While previous overview studies have documented the beneficial effects of PCT in specific mathematical subject areas, such as geometry, proportion, ratio, and arithmetic (Clements & Sarama, 1997), we still lack insights into the connections mathematics teachers make between PCT and mathematics, and how they experience them. It is highly likely that curricular decisions on what PCT areas to combine with what mathematical content will influence the connections pursued by teachers. In that respect, England provides an interesting case, since computing has been introduced as a distinct subject with its own curriculum (that closely relates to PCT). This implies that English mathematics teachers are not required to teach PCT as part of their mathematics teaching. In this paper, we study English teachers' current practices of teaching PCT as part of their mathematics teaching. Empirically, we rely on a survey sent to teachers in Denmark, Sweden and England, but in this paper, we exclusively report on data from respondents in England. Studying their approaches to teaching PCT and mathematics allows us insights into the extent to which teachers in England find PCT relevant to teach and, if so, in what ways they teach it. This leads us to ask the following research question:

What mathematical subject areas have been outlined as prominent to couple with PCT in research on mathematics education, and how does this correspond to how mathematics teachers in England include PCT in their teaching?

To answer this question, we begin by summarizing existing literature on which mathematical topics have historically been found to benefit from PCT. We then describe how different goals in different contexts and teacher-specific teaching choices seem to have an impact on how PCT and math are taught. We then describe what underlying construct of PCT we relied on for the survey data that informs this study. From this, we present our results regarding the use of PCT in mathematical subject areas by teachers in England. In the conclusion, we discuss the findings and reflect on the limitations of this study.

RELATED WORK

The idea of a close connection between PCT and mathematics is far from new in mathematics education research. In the 1970s and 1980s, Papert (1982) linked computational thinking and mathematics education, arguing that computers and programming can provide useful ways for students to interact with and learn mathematics. Concepts that were perceived as abstract and difficult to understand could, by drawing on computational thinking, be taught and understood in new, concrete, and systematic ways (Papert, 1982). Later, Wing (2006) emphasized the close relationship between computer science and general education, suggesting that computational thinking should be thought of as a general and basic skill similar to reading and writing.

Research on the potentials of PCT

While both Wing and Papert's work led to an interest in the combination of PCT and mathematics, the connection has been drawn to different mathematical topics. In particular, research found PCT to be beneficial in the teaching and learning of geometry, numbers, algebra, and arithmetic (Benton et al., 2018; Clements & Sarama, 1997; Noss, 1986; Papert, 1982). Prior to the work by Wing (2006), functions have also been thought to form a tight connection between programming and mathematics, as formulated in APOS theory (Arnon et al., 2014; Ayers et al., 1988; Dubinsky, 1984). APOS is an acronym for Action, Process, Object, and Schema. Actions are concrete, physical, or mental engagements with mathematical concepts and problems. Processes refer to mental operations or algorithms that students use to solve problems and understand concepts. Objects refer to mathematical concepts and entities that students encounter and learn about, such as equations, functions, or geometric shapes. Schemes refer to the organization and structure that students give to the concepts and processes they have learned. For example, understanding how to represent linear equations as functions in a coordinate system would be a schema. Using APOS theory, Dubinsky and colleagues (Ayers et al., 1988) explored how students develop their conceptions based on specific actions in a programming language and the ability to encapsulate mathematical processes with a programming language.

Research on the role of teachers in exploiting potentials of PCT

As the implementation of PCT into mathematics has become more common, interest in how it is implemented in the classroom has increased. The importance of the role of the teacher as a facilitator to bridge the connection between PCT and mathematics is evident from the ScratchMaths project, which consisted of a 2-year intervention for students aged 9 - 11 in England with a distinct focus on algorithms. In this project, Benton et al. (2018) found differences in the students' learning from the intervention related to differences in whether teachers discussed algorithms as a general phenomenon or as different, comparable "strategies" throughout the intervention.

Misfeldt and colleagues have investigated Swedish mathematics teachers' conceptions of the relation between mathematics and programming through a survey of 133 Swedish teachers following an in-service training initiative (Misfeldt et al., 2019). Misfeldt and colleagues found that the teachers identified a strong – but not very strong – link from programming to mathematics, that they were interested in working with programming, but that the teachers did not feel ready to do so.

These studies illustrate that the way teachers choose to implement PCT in their math teaching is important for what their students get out of it, and that teachers do not necessarily feel ready to teach programming, even if they do see a link between that and mathematics. Thus, it shows the need for understanding not only how connections between programming and mathematics are identified theoretically, but also how these two are connected (in the classrooms) by teachers. This paper takes up the mantle of such an approach by conducting a descriptive analysis based on teachers' answers to a survey to examine how math teachers in England apply PCT in relation to different mathematical topics. Again, teachers in England provide an interesting case, due to the national implementation of PCT in a subject separate from mathematics, which in principle should leave teachers to only implement it in mathematics when they find good reasons to do so.

THEORETICAL FRAMEWORK INFORMING THE DATA COLLECTION

As argued above, we have better insight into the potential of PCT for teaching and learning specific mathematical topics than into what mathematics teachers actually do. Our study therefore sought to allow the respondents themselves to report in which subject areas of mathematics they teach PCT. This design of our survey is informed by two theoretical sources: a definition of PCT, on which we have based the survey, and a conceptualization of mathematical topics.

First, we defined computational thinking for the respondents of the survey to ensure a common understanding of what it entails. For this, we relied on the definition by Kallia et al. (2021) of a structured problem-solving approach, namely: "A structured problem-solving approach in which one is able to solve and/or transfer the solution of a mathematical problem to other people or a machine by employing thinking processes that include abstraction, decomposition, pattern recognition, algorithmic thinking, modelling, logical and analytical thinking, generalisation and evaluation of solutions and strategies" (Kallia et al., 2021, p. 28). This allowed computational thinking to be understood in relation to how it could be integrated in mathematics.

Second, we wanted to describe distinct subject areas in which the teachers could use PCT to teach mathematics. Here, we relied on the 10 subject areas described in the KOM framework developed by Niss and Højgaard (Niss & Jensen, 2002; Tamborg et al., 2022). These areas include numbers, arithmetic, algebra, geometry, functions, infinitesimal calculus, probability, statistics, discrete mathematics, and optimization. By drawing on this categorization and combining it with the definition offered by Kallia et al. (2021), we are able to examine the relationship between PCT and its use by the respondents. While the potential of combining specific PCT and mathematical topics can fluctuate over time and tend to dwell on specific themes and subjects, looking at what the teachers do in their own practice might enable us to see what the teachers view as feasible connections between PCT and mathematics.

DATA & METHOD

The analysis builds on data from a comparative study of how PCT is integrated with mathematics teaching at the compulsory level in three countries with design experiments and the development of exemplars (Tamborg et al., 2022). In this article, we only include data from teachers in England. The survey was initially distributed to a random selection of schools from the English government's website. It was sent out to the school leaders, who were contacted multiple times over a three-week period. As there was a limited response from the schools, we obtained additional answers by sending a link out to 300 teachers through a central gatekeeper in England, who urged these teachers to share it with their colleagues. This network is closely affiliated with initiatives to use technology when teaching mathematics in England. This is likely to introduce a selection-bias, with the teachers who answered our survey being more inclined to use technology (and potentially PCT) in their teaching than their average colleague.

Who answered the survey

108 teachers in England answered the survey, and 62 of them provided answers to questions about their use of PCT. The teachers taught classes with students between 12 and 16 years old, which translates to lower secondary school. As mentioned in the introduction, England provides an interesting case as the computing curriculum in this context is separate from mathematics. There are thus no curricular requirements for the respondents to cover PCT as part of their mathematics teaching. Despite the fact that the English school system has included a computing curriculum since 2013, only half of the teachers reported that their students have a course in computer programming or computational thinking. 25 percent reported that such a subject is not taught at their school, while another 25 percent were unsure, but we do not know whether the respondents' students have been taught computing at another point during their schooling. Furthermore, only nine of the 62 teachers allow their students to use computers in their classes, and six of the teachers work at schools that do not allow computers at all.

The teachers partook to a very limited extent in formal professional development on both computer programming and computational thinking. Only six teachers thus received training in computer programming over the last two years, while 20 teachers participated in some professional development in computational thinking. 15 of the teachers have informally (e.g., through YouTube videos) trained themselves on computer programming, and 23 of the teachers have done the same for computational thinking.

Asking about subject areas and PCT

In order to see the connections made by teachers between subject areas and PCT, we first show the share of teachers who are employing PCT and then what subject areas they are covering with PCT. The teachers' use of PCT comes from the question: "Are there elements of computer programming or computational thinking in your mathematics teaching? (select all that apply)". Here, the survey distinguishes between computer programming and computational thinking, which were defined for the respondents as described in the prior section. Furthermore, the survey included a distinction between computational thinking being taught by digital means (e.g., by using a computer programme) and computational thinking being taught by analogue means (e.g., by making the students draw operations). The teachers could also respond that they did neither or that they did not know whether they did so.

The teachers who do include PCT in their teaching are asked in what subject areas they use PCT by answering the question, "In which of the following mathematical subject areas do you include computer programming in your teaching?" The teachers were presented with the 10 subject areas described in the section on the theoretical framework. In the survey, the respondents were also asked to report to what extent they use PCT to cover the subject. Due to the limited number of respondents compared to the complexity of our analysis, we only compare whether it was used or not to cover the subject. Furthermore, we investigate in what mathematical subject area the respondents taught PCT.

RESULTS

Of the 62 teachers who answered the questions about using PCT in their teaching, 25 do not use PCT in their teaching, while six were unsure whether they used PCT in their teaching (Figure 1). 23 of the teachers indicated that they use analogue computational thinking in relation to their teaching, the largest group indicating use of PCT by far, while only 10 and 6 teachers use computer programming and digital computational thinking, respectively. Analogue computational thinking thus seems to be the most common PCT element.

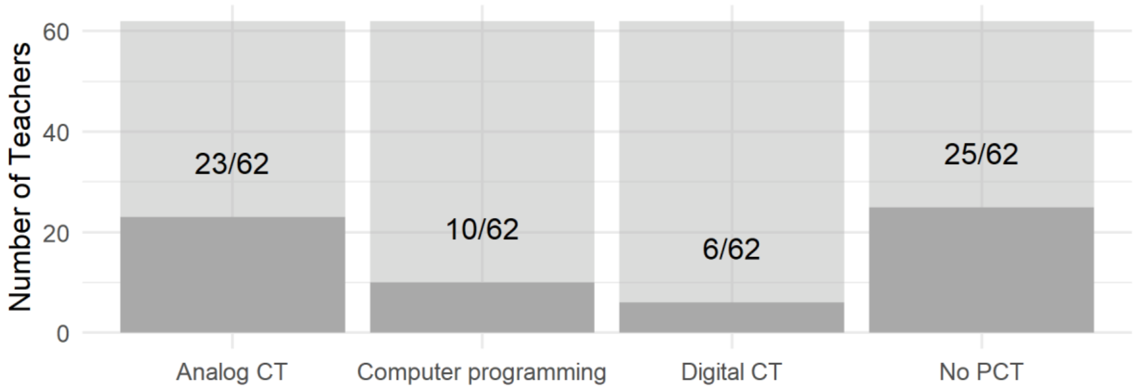

Figure 1. Teachers' use of PCT in their teaching

Figure 2 shows the distribution of teachers who use PCT in relation to the ten different areas of mathematics. Again, it seems useful to look at this by compartmentalizing the different subject areas into groups dependent on the number of teachers who answer that they use PCT in relation to them.

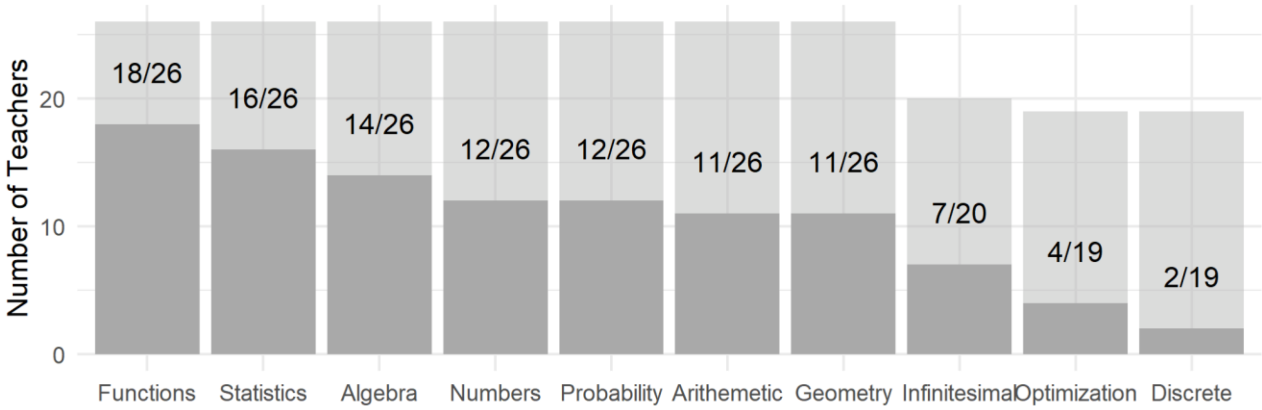

Figure 2. PCT in subject areas

We can thus locate three different 'groups' of subject areas, with a different number of teachers indicating use of PCT in relation to each. The subject areas to which most teachers apply PCT include functions, statistics, and algebra (18, 16, and 14 teachers, respectively). The areas that the second-most teachers indicate using PCT in relation to are arithmetic, geometry, numbers, and probability (12, 12, 11, and 11 teachers, respectively). Infinitesimal, optimization, and discrete mathematics has the most limited connection to PCT as counted by teachers' use (7, 4, and 2 teachers). Multiple teachers furthermore indicate that these final three subject areas are not part of the mathematics curriculum.

Finally, figure 3 shows which combinations of subject areas teachers indicate they use PCT in relation to. Though we cannot know for sure whether the teachers are actively combining these areas, it might point towards trends in that regard. The model shows the top ten most frequent subject area combinations and the number of times respondents noted that they had covered both by PCT. The most common combinations of subjects are between statistics and functions, and between algebra and functions (13 teachers). Next are the combinations of functions and arithmetic, functions and probability, and probability and statistics (11 teachers).

Besides being the most commonly used ‘code’, functions are also most closely connected to the other subfields, and specifically to algebra and statistics. Statistics are interesting, as they are less well-described as having a strong relation to computational thinking as e.g., geometry or numbers would have. Geometry is surprising due to its significant role in the literature and its absence both as a highly integrated field and as a less common combination with the other subject areas.

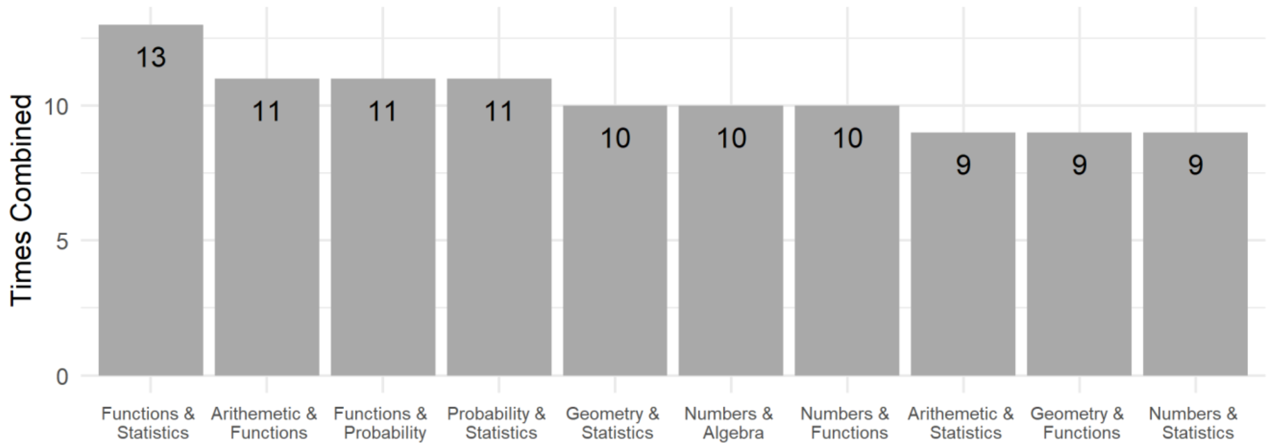

Figure 3. Subject areas combinations covered by PCT

DISCUSSION AND CONCLUSION

In this paper, we studied to what extent mathematics teachers in England teach PCT and in what mathematical subject areas they do so. Thus, we try to bridge studies of the mathematical potentials inherent in including PCT with studies of how teachers are employing PCT in their classrooms. Looking at responses from 62 mathematics teachers in England, we found that about half of them are applying one or more components of PCT to their teaching. A majority of this is done by analogue computational thinking. It is likely that this reflects the fact that the majority of the teachers who took part in our survey indicated that they do not allow computers in their classrooms. As our sample might suffer from a selection-bias towards more tech-prone teachers, this finding can most likely be generalized to the English population in general.

The teachers who teach PCT as part of their mathematics teaching most often do so in the subject areas of functions, algebra, and statistics. While algebra and functions are highlighted in the state-of-the-art, statistics is less commonly associated with PCT. When comparing the teachers’ choice of subject areas to teach PCT, there are two surprises in comparison with the state-of-the-art. First, the most common connection between PCT and functions has been more or less forgotten (or at least left out) in recent research. Second, while geometry is commonly described in the state-of-the-art as being much used with PCT, only 11 of 26 teachers report it as part of their practice. A reason for this discrepancy could be the focus on analogue uses of computational thinking in England, which is in opposition to the focus on digital platforms and digitally enforced computational thinking in the literature. Thus, if there is a digital precedence for including PCT in geometry, then the learning materials might include digital remedies. If teachers do not include digital materials (as the case is for many of the teachers in England), then they will have access to less materials to include. This is however rather speculative, as it does not explain why the specific subject area geometry would be more affected by the analogue teaching.

The limited number of respondents does not allow us to test the statistical significance of the findings. However, we believe that outlining what subject areas teachers are applying PCT to is an important first step toward understanding where teachers see potential for such integration and raises some important questions. One of these is whether the distinction between digital and analogue ways of engaging with PCT is impacting the potentials in relation to engaging students with different mathematical subject areas.

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