STUDENTS' MEANING-MAKING PROCESSES IN THE DIGITAL ERA: HOW CAN THE TEACHER FOSTER MATHEMATICAL DISCOURSE?

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The aim of this work is to study how the teacher can prompt students' meaning-making processes in a technology-rich context. With this purpose, we present a teaching activity on the notion of rotation as an isometry of the Euclidean plane involving a dynamic geometry environment. The experimentation of the teaching activity in a 7th-grade class has been analysed and discussed taking into account the notion of teacher instrumental orchestration and elements of Sfard's commognitive framework to show how the teacher prompted the evolution of the students' mathematical discourse. Although the paper presents and discusses only an example of mathematical discourse during a collective discussion orchestrated by the teacher, it aims to contribute to the reflection on the value of educational practices in an era of intense digital transformations.

Keywords: Mathematical discourse, Instrumental Orchestration, Commognition, Dynamic Geometry

INTRODUCTION

Technologies have recently assumed an increasingly important role in everyday life, as well as in the school context, and their use has influenced the way teaching activities are carried out in the classroom. Hence, growing relevance has been assumed by the teachers' need to rethink their practices in order to effectively and consciously make sense of the integration of technologies into teaching activities. As technologies can offer teachers opportunities to create appropriate learning environments in which students are involved in the construction of mathematical meanings, their integration has become one of the main research topics in mathematics education (Trgalová et al., 2018). This issue is approached by taking into account several perspectives: the design and development of resources; the development of the mathematics curriculum with an appropriate design of activities and/or sequence of activities; and the benefits that can be gained by exploiting the potential offered by technologically rich environments in a way that fosters students' learning. In particular, following the discursive approaches to research in mathematics education (Sfard et al., 2001), the use of technology in the teaching-learning processes can be investigated for its role in fostering the production of mathematical discourses rich in conjectures and in the need to verify them.

In this paper, we are interested in reflecting on how the evolution of students' mathematical discourse in the meaning-making processes can be supported through appropriate activities involving technologies and conscious teacher behaviour. To this end, we present and discuss the results of a teaching activity on rotation involving a dynamic geometry environment with the aim to study the students' mathematical discourse and the way the teacher can prompt its evolution. The notion of teacher instrumental orchestration developed in the field of research in mathematics education and particularly in the case of the use of technologies in the classroom (Trouche, 2004; Drijvers et al., 2010) will be our reference to analyse the role of the teacher. The analysis of the data gathered (video recordings, transcripts and students' protocols) has been developed taking into account Sfard's (2001, 2008) commognitive framework concerning the role of interpersonal communication in mathematical thinking and learning.

Through an example of the analysis of the evolution of the mathematical discourses, results will show how the teacher's orchestration of the activity led the students to collectively construct the meaning of rotation by exploiting the potential of dynamic geometry.

THEORETICAL FRAMEWORK

The Instrumental Orchestration

Starting from the metaphor of the orchestra as a harmonic composition of different instruments, Trouche (2004) offered a theoretical lens to describe how the teacher can coordinate different coherent sets of artefacts within the classroom, with the aim of guiding the students' instrumental genesis (Artigue, 2002) thus improving the teaching-learning process. Drijvers and colleagues (2010) then paid their attention to the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment in a given mathematical task situation, distinguishing three main elements: a didactical configuration, an exploitation mode and a didactical performance. Referring to the metaphor of musical orchestrations: the didactical configuration can be compared to the choice of musical instruments with which to compose the group and the choices about their arrangement in space so that the different sounds produce polyphonic music; the exploitation mode can be compared to the determination of the partitioning of the music for each of the musical instruments involved, taking into account the expected harmonies that will emerge; the didactical performance can be compared to the musical performance, in which the actual interaction between the conductor and musicians reveals the feasibility of the intentions and the success of their realisation. In the next sections, to describe how the teaching activity analysed in our study was orchestrated by the teacher, we will present the didactical configuration, the exploitation mode and the didactical performance.

The Commognitive approach

Extending the concept of thinking from the individual sphere to the interpersonal sphere, Sfard (2008) defines thinking as the individualised form of communication activity. Thus, thinking stops being separate from any communicative act and becomes, together with cognitive processes, the same representation of a specific phenomenon. Therefore, Sfard (2008) combines the terms cognition and communication, producing the new term *commognition* and illustrating how this approach can be applied to mathematical thinking: unlike other scientific or colloquial discourses, mathematical discourse is characterised by objects that are discursive constructs and part of the mathematical discourse itself. Accordingly, mathematics is seen as an autopoietic system, comprising both the discourse and its objects, which is therefore capable of growing incessantly from within as new objects are added (Sfard 2008, p. 129). Therefore, mathematical discourses are described through the *words* (i.e., the key terms used to describe their characteristics) and the *visual mediators* (i.e. the artefacts on which the communication process operates). Indeed, various objects are involved in mathematical discourse, such as signifiers (i.e. the word or symbol that functions as a noun in the discourse participants' utterances), realizations, primary objects and discursive objects. Realizations can take the form of written or spoken words, algebraic symbols, drawings, or even gestures. While a signifier can be considered as a "root" of a "*tree*" *of realizations*, which Sfard defines as a "*hierarchically organised set of all the realizations of the given signifier, together with the realizations of these realizations, as well as the realizations of the latter realizations and so on*" (Sfard, 2008, p. 300). Realization trees and, thus, mathematical objects are personal constructs and provide important information on the evolution of a given person's mathematical discourse. Hence, taking into account the form and results of their processes, mathematical discourses are characterised by the *endorsed narratives* they produce and the mathematical *routines* (Sfard, 2008, p. 245) which follow the process of creating and validating the narratives. From this perspective, mathematical routines can be regarded as explorations in that they end with narratives that contribute to the creation of a mathematical theory. Moreover, the *ritual* is also an important component of learning mathematical discourse and is realised as a willingness to be part of a certain community and to speak the same language. In other words, ritual represents the unconscious production of actions that can only simulate a certain acquisition of knowledge and skills by students. Therefore, the teacher's purpose is to transform ritual into exploration in such a way that students become aware of their own mathematical activities and discourses, thus evolving them towards constructing mathematical meanings.

Research question

With the aim to contribute to the reflection on the value of educational practices in an era of intense digital transformations, in this work, we analyse the results of a teaching activity on rotation involving a dynamic geometry environment attempting to answer the following research question: how can the evolution of students' mathematical discourse in the meaning-making processes be supported through the teacher's orchestration?

METHODS

This study is based on the analysis of the video recordings of a teaching activity (consisting of four phases), developed and experimented with by an expert teacher in her 7th-grade class, aiming at the construction of the notion of rotation as an isometry of the Euclidean plane. To investigate how the evolution of students' mathematical discourse in the meaning-making processes can be supported through the teacher's orchestration, we first read the development of the teaching activity in terms of didactical configuration, exploitation mode and didactical performance. Moreover, according to the learning aim of the designed activity, and with reference to Sfard's framework of Commognition, in order to answer our research question, we created an example of a rotation signifier realization tree. Then, the transcripts of the videos were analysed, using our realization tree, attempting to study the evolution of the students' mathematical discourse on the rotation signifier. In this paper, we discuss an example of such analysis. In what follows we start presenting an overview of the four phases of the teaching activity focusing on the main elements of the teacher's orchestration.

Instrumental orchestration of the teaching activity

The *didactical configuration*, characterised by the involved artefacts and the teaching setting, is defined by: the synergic use of a manipulative artefact and a dynamic geometry environment (GeoGebra); the students' individual use of a digital device (iPad); and finally the use of an Apple TV that allowed the sharing of any of the participants' screens. The teacher appreciated the idea of synergically combining the use of different kinds of artefacts (Faggiano et al., 2018), and more precisely to introduce the topic by means of a manipulative tool. She then made her own choices to adapt the sequence (that was designed by pre-service teachers and researchers (Faggiano & Mennuni, 2020)) to the context of her class. In particular, she made some changes in the choices of the artefact to be used in the second phase (that was previously designed to be accomplished using paper, pencil, ruler and compass), asking students to realise the requested construction of a rotated figure using their iPads and the tools in GeoGebra. This was due to the students' familiarity with GeoGebra and her usual mode to exploit the affordances of the Apple TV to share students' work during the discussions.

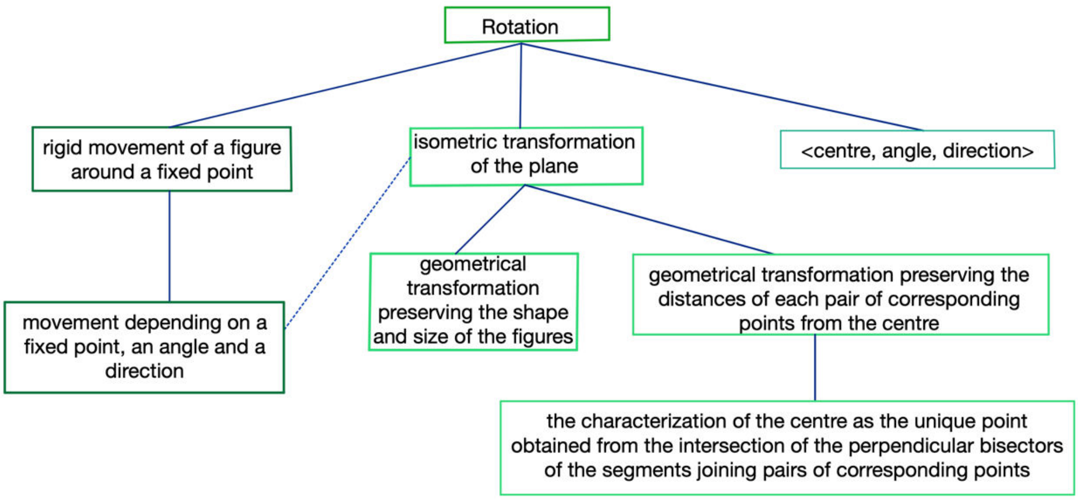
The *exploitation mode* is described by the design of the teaching activity with its four phases. In each of the four phases of the activity, students are asked to accomplish a task using the given artefacts. Then a collective discussion is orchestrated by the teacher to focus students' attention on the specific aims of the phase. During the first phase, students are involved in characterising rotation as a rigid movement dependent on an angle and a fixed point called the centre, through the use of a manipulative tool. In this phase, students are prompted to identify an initial idea of the conservation of distances between the centre and each pair of corresponding points on the two flags. In the second phase, GeoGebra is used with the aim of pushing students to build rotated figures by exploiting the dynamic geometry potentials and the observations done during the previous phase. In the third phase, students are given a GeoGebra file containing a flag, a point P and a slider α representing the angle. They are asked to use GeoGebra's Rotation tool to construct the rotated flag (with respect to P and the angle α). Then, students are asked to describe what happens, and to explain the reasons, if they change: (a) the angle by moving the slider, (b) the point P, (c) the position of the initial flag. Finally, in the fourth phase, students are given in GeoGebra two rotated flags and they are asked to find the hidden centre of rotation. The aim of this phase is to characterise the centre as the unique point obtained by intersecting any two perpendicular bisectors of every segment joining each pair of corresponding points. This requires the use of the property, as it arose in the previous phase, regarding the preservation of the distances between the centre and every pair of corresponding points.

The *didactical performance*, which is characterised by the interactions among the teacher, the students and the artefacts, reveals the students' construction of the meanings. The entire teaching activity required four hours and involved a 7th-grade class. At the beginning of the teaching activity, the teacher explained to the students that they would have to use a manipulative tool and their iPads to accomplish four different tasks (related to the four phases described above), to be discussed collectively at the end of each phase. The GeoGebra files and worksheets of the tasks have been shared with the students through the AirDrop function on their devices. During each phase, students were given 20 minutes to accomplish the different tasks, and the teacher used the remaining 40 minutes for the collective discussion. We do not have enough space to report the exact details of the tasks given to the students, but the teacher's requests were in tune with the exploitation mode. Herein, we focus on the discussion that followed at the end of the third phase of the activity. To help the students construct the properties of the rotation, the teacher asked them to share their GeoGebra file on the Apple TV in the classroom and explain their observations. To guide the students in constructing the realization of the rotation signifier related to the property of preservation of the distances of the corresponding points from the centre P, they were pushed to interact directly with the tools on GeoGebra. In particular, the teacher asked the students to activate the trace on points A and A' and describe what they observed when changing the values of the angle α with the slider. To achieve her aim, the teacher asked the students to draw the segments AP and A'P and pushed them to observe that the trajectories described by the traces of points A and A' were circumferences with the centre in P and consequently the distances between P and the points A and A' had to be equal since they were the radius of the circumferences.

In this paper, to bring to the fore the role of the teacher's instrumental orchestration, we analyse the meaning-making process during the collective discussion conducted by the teacher at the end of the third phase of the teaching activity.

RESULTS AND DISCUSSION

As a reference to investigate the evolution of the students' mathematical discourse, considering the learning aims of the teaching activity, we have built the realization tree in Figure 1. In the leftmost branch, we started with the idea of rotation as a rigid movement that is the focus of the first two phases of the activity with the manipulative tool. This first realization can be expanded by pushing the students to observe that with this movement, the flags do not change their shape and size and that the movement only depends on a point, an angle and a direction of rotation. The most important part of this realization tree consists of the idea of rotation as isometry. In tune with the aims of the third phase of the activity (on GeoGebra), this realization can be expanded considering the property that characterises the centre of rotation, i.e. the preservation of the distances of each pair of corresponding points from the centre. This last realization can also evolve, exploiting the potential



of GeoGebra's tools as it is in the design of the fourth activity, in a further realization of the characterization of the centre as the unique point obtained from the intersection of the perpendicular bisectors of the segments joining pairs of corresponding points. Finally, for the sake of completeness, we have also included an essential realization of the rotation signifier that only considers rotation as a plane transformation that can be described by the triad (centre, angle, direction of rotation).

Figure 1. Our realization tree of "rotation"

In this paper, we focus on the realization of the rotation signifier by considering the construction of the narratives concerning the preservation of the distances between the centre and the pairs of corresponding points. To show the evolution of the students' mathematical discourse on the rotation signifier as it was fostered by the teacher's orchestration of the collective discussion, we analyse two excerpts taken from the third phase of the activity. These excerpts concern the discussion on what happened to the rotated flag when changing the angle of rotation through the slider.

1	Teacher:	I would like to make you think about how A and A' vary, how B and B' move between them. I don't know if you saw those red dots which were there. *[With her fingers she traces in the air the trajectory of points]*
2	Teacher:	I am interested in whether they move at will or they are in some way constrained. Maurizio?

		[She moves her hands randomly]
3	Maurizio:	They move with respect to the distance from point P
4	Teacher:	They definitely move according to distance, and how are these distances? *[She sharply points out "definitely". To talk about the distance she fixes a segment with her fingers]*
5	Maurizio:	They are the same
6	Teacher:	What is the same?
7	Maurizio:	The two distances of the real flag and the clone flag from point P are the same *[To describe this dependence between the two flags and their relative distances from the centre, the student moves his right forearm downwards. And at the same time, the teacher marks the distances PA and PA' on the screen]*

This first excerpt refers to the moment in which the teacher decided to push the students to think about and reflect on the movement of the pairs of corresponding points. In her intervention reported in [1], the teacher intentionally led students' attention to the GeoGebra trace of the points of the rotated flag. Her intervention came with a small gesture made with her fingers to describe the movement that the red dots drew on GeoGebra. At this point, the student Maurizio answered (in [3]) the teacher by creating his own narrative to be validated by the teacher: he started with the idea of the distance from the centre of rotation, which is a new realization of the rotation signifier. The teacher felt that Maurizio's mathematical discourse could be guided to evolve towards the identification of the trajectory of the movement as a circumference. Accordingly, the teacher approved his narrative (in [4]) also with a specific gesture: she fixed a segment with her fingers and moved it back and forth twice with respect to a fixed point. When the teacher asked him what these two distances looked like, Maurizio replied that "they are the same". The teacher then decided to share the GeoGebra file with her iPad on the Apple TV –in which the measures appeared along the distances considered– and asked (in [6]) for further clarification: she wanted to understand if Maurizio's answer was due to visual observation alone - and in this case, according to Sfard's theoretical framework from his answer we find a ritual - or if it was a new narrative originating from some exploratory mathematical routine that therefore also needed to be approved. The answer in [7] given by Maurizio resolved the teacher's doubt. Maurizio's intervention, "*The two distances of the true flag and the clone flag from point P are the same*", shows how he has abandoned the passive use of the word rotation introduced by the teacher from the beginning. Indeed, this intervention makes it clear that this student constructed his own realization of the signifier rotation: the rotated flag is realized with the expression "*clone flag*". So from this point, it can be seen how Maurizio's mathematical discourse on the rotation signifier was evolving as he was constructing his personal realization of the rotation. Consequently, it is possible to find a link between the intervention of the same student reported in [3] and that in [7] insofar as through the word "*clone*", Maurizio emphasised the idea of the dependence of the rotated flag on the initial flag, but also on the centre of rotation. Maurizio's evolution of his mathematical discourse depended on the teacher's orchestration of the collective discussion. In fact, the teacher's attention was attracted by the word "*clone*" and the linked expression "*the two distances between...*" so she decided to interact with the artefact marking the distances on the screen and pointing her index fingers at the two extremes of the segments (in [7]).

Thanks to the orchestration and the teacher's interventions, the other students in the class also seemed to have enriched the rotation signifier with the new realization linked to the preservation of the distances of the corresponding pairs of points from the centre.

8	Teacher:	Let's see Ilaria
9	Ilaria:	I have joined A and its correspondent [A'] and they trace a trajectory, a circumference centred on P *[She joins her palms together]*
10	Teacher:	They draw a circle centred on P. Is this always true?... do you agree with Ilaria?
11	Ilaria:	I have done the same thing for point D

What is shown in this second excerpt is the narrative that Ilaria exhibited to the teacher after a brief confrontation with her classmate Maria (who traced with the pen in the air a circumference). In fact, from the analysis of the video recordings it was possible to observe that the evolution of Ilaria's mathematical discourse took place from a narrative between the two students, characterised mainly by glances and gestures. This confrontation between the two students arose from the stimuli given by the teacher during the discussion. Invited (in [8]) by the teacher (who has noticed the interaction with Maria), Ilaria presented her narrative reported in [9], accompanied by the gesture of joining the palms, to have it validated by the classmates and the teacher. As Ilaria took a single pair of corresponding points and built the circumference centred in P, with her intervention reported in [10], the teacher repeated the construction made by Ilaria (with Maria) and tried to stimulate all the other students, and in particular Ilaria, to go over. Indeed, with the intervention in [11], it is clear that Ilaria was able to generalise the property for the other pairs of corresponding points. This excerpt can also be interpreted by exploiting the aspects of mathematical discourse introduced by Sfard. Indeed, in this case, we are dealing with a mathematical routine that can be characterised as follows. There is first a *ritual* in which Maria and Ilaria must confront each other before exposing their narratives to the teacher. This example can be seen as a ritual in that this behaviour of the two students was found several times during the activity and especially during the collective discussions, demonstrating a certain iterated character in itself. In addition, this behaviour highlights the two students' need to create a mathematical discourse in order to discover the object under investigation, which must pass through validation of their own peer narrative. Then, it becomes a mathematical *routine* of an exploratory nature in that the narratives of the two students are subsequently endorsed by the teacher, contributing decisively to the discovery of the new realisation of the signifier.

Guided by the teacher, the students needed to describe the movement of the flag's points as the position of the initial flag moves with a circumference with centre P to which both the points of the initial flag and those of the rotated flag belong. This additional realization of the rotation signifier is fundamental in the evolution of the students' mathematical discourse. It will allow them to enrich the realization of the rotation constructed up to that point. This enrichment is essentially given by associating the distance between the centre of rotation and the corresponding pairs of points on the flags with the radius of the circumference that has just been identified.

CONCLUSIONS

With the attempt to investigate how, in a technology-rich context, the teacher can foster the evolution of students' mathematical discourse in the meaning-making processes, this study is framed by the notion of the Instrumental Orchestration and the Commognitive approach.

Through the analysis of our teaching experiment, we have shown that the teacher's orchestration of the activity and the related collective discussion led the students to collectively construct the realization of the rotation signifier related to the conservation of the distances of the points of the two flags from the centre of rotation and consequently pushed the students towards an evolution of their mathematical discourse.

Although the paper presents and discusses only an example of mathematical discourse during a collective discussion orchestrated by the teacher, it aims to offer a contribution to reflect on the value of educational practices in an era of intense digital transformations. Indeed, results are representative of how the teacher's orchestration can guide the students' development and evolution of mathematical discourse, thus fostering the students' meaning-making process.

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