A CROSS-NATIONAL COMPARISON ON HOW THE CONGRUENCE AND SIMILARITY OF FIGURES IS ADDRESSED WITH TECHNOLOGY

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In this paper we briefly examine how digital technologies are included in the curriculum guidelines of two countries in Europe (Italy and Portugal) and two in Latin America (Colombia and Mexico) and present four examples of how technology can be very differently used to address the topic of congruence and similarity of figure at middle school level. This work is part of a cross-national study in which we focused on the factors affecting digital technology (DT) integration in mathematics education. The aim of this paper, that is not a research paper and is far to be comprehensive or generalizable, is to provide insights into the challenges of DT integration in mathematics education, drawing the attention of the readers to the impact and limitations that various factors can have on it.

Keywords: Technology integration, Cross-national comparison, Congruence and Similarity of Figures

INTRODUCTION

Although national policies and curricular recommendations promote the use of technology in mathematics education, digital technology (DT) resources are often underutilized in practice. Research (e.g. Julie et al., 2009; Sacristán, 2017; Sacristán et al., 2021; Faggiano et al., 2021) has also shown that in many countries, rather than utilizing mathematics-enhancing digital resources, technology tends to be used as an add-on to traditional paper-and-pencil tasks and/or for presentation and communication purposes (which was exacerbated by the COVID-19 pandemic).

The integration of DT into mathematics education is influenced by a range of factors at various levels, including societal and governmental policies, regional and school-level regulations, and teacher practices. At the teacher level, for example, challenges can arise due to the absence of clear guidelines in official documents and curricula regarding the appropriate utilization of recommended resources or how to determine when to employ them.

In this work, through a cross-national analysis (Jablonka et al., 2018), we aim to show very different approaches to how technologies are conceived and used. Specifically, we examine how digital technologies are included in the curriculum guidelines of two countries in Europe (Italy and Portugal) and two in Latin America (Colombia and Mexico), and present examples from each country of how one middle-school mathematics topic is addressed using DT resources.

We selected a topic that is present in all of our middle school curricula, which we deemed suitable for being taught using digital tools, specifically dynamic geometry. The topic we chose was *Congruence and Similarity of Figures*. Surprisingly, we found very few examples of digital approaches to this subject in materials designed for teachers, with the most notable ones originating from Italy and Mexico, the latter being quite dated. We begin by exploring the Italian approach, which we regard as the most thorough.

THE CASE OF ITALY

In the introduction of the Italian National Guidelines for basic school (MIUR, 2007) that state the goals and aims for mathematics, there are some general considerations regarding the use of technology. The guidelines underline that it is important to encourage the deliberate and purposeful use of calculators and computers beginning in the early years of primary school. This may include using these tools to verify the accuracy of mental and written calculations, as well as to explore the world of numbers and shapes. The use of technology is once again highlighted in the list of the learning aims to be achieved by the end of middle school, specifically in relation to the themes of "numbers" and "space and figures". Students should learn to: perform basic mathematical operations between different types of numbers (natural numbers, whole numbers, fractions, and decimal numbers) using mental math, standard written algorithms, calculators, and spreadsheets, while carefully considering which tool is most appropriate for the task at hand; reproduce geometric figures and drawings with precision and accuracy, utilizing appropriate tools such as rulers, squares, compasses, protractors, and geometry software. However, it is worth noting that, although there are official ministerial projects on coding and computational thinking in Italy, these are not curricular and mathematics is not mentioned at all (e.g., see https://www.indire.it/en/progetto/computational-thinking/).

Additionally, there are some valuable, yet often overlooked, resources available on official websites such as from the Italian Mathematics Union (UMI-CIIM: https://umi.dm.unibo.it/materiali-umi-ciim/) or in the final reports of national projects like PQM (2015) and mat@bel (http://www.scuolavalore.indire.it/superguida/matabel/). These resources have been developed by a team of experienced teachers to support the improvement of learning outcomes in the logic-mathematics field. Most of these resources are designed for middle and secondary school students and sought to harness the potential of new technologies. The following example comes from the repository of the PQM ministerial project.

A teaching activity on proportionality with dynamic geometry at 8th grade

The teaching activity we present concerns proportionality in Geometry and is based on the use of dynamic geometry (GeoGebra) to introduce 8th-grade students to the concept of Similar Triangles and their properties (http://www.scuolavalore.indire.it/nuove_risorse/proporzionalita-in-geometria-con-geogebra/). Figure 1 shows the structure of the teaching proposal that is designed to be developed through five activities/tasks (including the brief introduction to some relevant GeoGebra functionalities) and to last around twelve hours (including the final assessment and a further remedial or supplementary activity). Methodologically it is suggested that pupils should work in pairs using one worksheet per pair in order to encourage initial peer discussion on the proposed activities. These activities are primarily heuristic and generally follow the following sequence: first, analyzing or reproducing a drawing, followed by structured reflection on the assigned task, then discussion with the teacher on proposed solutions, and finally, the teacher providing a definition for the introduced concepts. The teacher plays a crucial role in managing student reflections and comments, effectively facilitating discussion and ultimately arriving at a shared definition of the concept to be introduced.

The primary goal of the initial activity is to create a set of similar triangles that share a common angle and to establish a relationship between their corresponding sides. To begin, students will be asked to use the "dilation" tool and to make conjectures about how it works. Next, students will participate in an exploratory activity that encourages them to create pairs of triangles and formulate their own conjectures regarding patterns that may exist between their side measurements. Through a teacher-mediated collective discussion, students will work together to formulate an accurate definition of the property they had previously conjectured. The teacher will introduce the term "similar" to describe the pairs of triangles that were constructed. Finally, the activity will culminate with the discovery of the properties of similar triangles, which will help to establish the concept of proportionality in geometry and provide a basis for exploring Thales theorem.

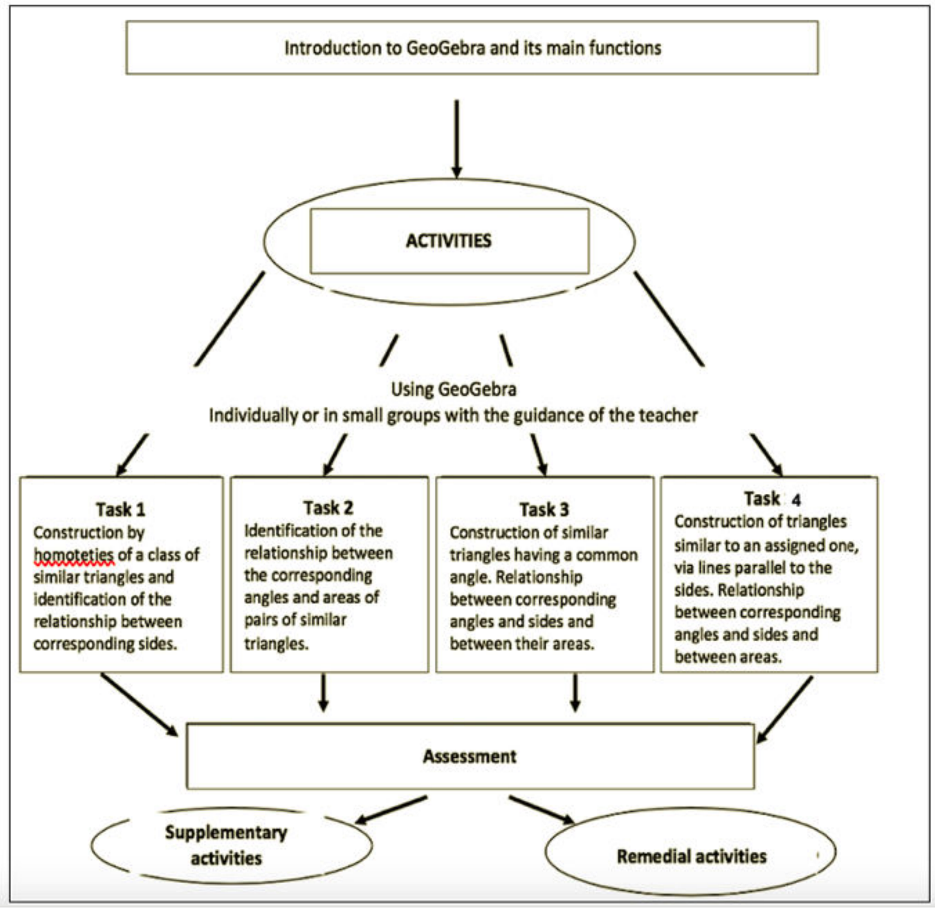


Figure 1. Structure of the teaching proposal on Proportionality in Geometry, with Geogebra. (Translated from PQM, 2015)

By utilizing dynamic geometry software, pupils can manipulate the constituent elements of constructed figures through drag-and-drop activities, resulting in both direct and indirect movements. The production of a conjecture is founded on the interpretation of invariants and the relationships between them. This makes it an excellent example to demonstrate how the dynamic nature of the software can play a critical role in enabling pupils to formulate and strengthen their conjectures, thanks to the drag-and-drop function.

THE CASE OF PORTUGAL

In Portugal, the official curricular documents clearly value the use of DT. They consider DT as unavoidable and powerful resources to promote more meaningful learning and expand the contexts in which the student's action takes place and the diversity of perspectives on the mathematical objects studied. And, since 2021, there have been curricular changes that include Computational

Thinking as part of the mathematics curriculum (implemented in 2022, although many teachers do not yet know how to address it). The curricular document (ME, 2021) values mathematical literacy, and defines the aims that all students must be able to achieve. The digital literacy of students ought to encompass a variety of skills, such as conducting calculations, creating graphs, running simulations, gathering and organizing data, engaging in mathematical experimentation, researching, modelling, and sharing ideas. The curricular document (ME, 2021) also includes more specific information about DT use for each mathematical topic. For instance, the emphasis on the use of technology is reinforced in the context of geometry teaching –"The study of geometric objects must be accompanied by experience (where technology plays a key role)" (pp. 10-11)– and there are several references to technology use in relation to the teaching and learning of the concept of similarity.

At present, the Ministry of Education has a collection of tasks available on its website (http://aem.dge.mec.pt/pt/recursos) designed to aid teachers. However, the majority of these tasks are in a traditional paper-and-pencil format, and only a limited number suggest utilizing DT. Currently, publishing companies are creating educational materials for teachers, which comprise books, also centred on computational thinking, featuring exercises that utilize tools like GeoGebra, Excel and Scratch. The following example comes from an official textbook (Almeida, 2022).

A task on similar triangles with dynamic geometry at 7th grade

In Figure 2, we present a task for 7th-grade students that requires them to create sets of triangles that meet specific criteria. The students are then prompted to scrutinize the pairs of triangles to identify similarities and experiment with different combinations. Based on their findings, the students are expected to develop an understanding of the conditions that two triangles must meet to be considered similar.

Similar triangles

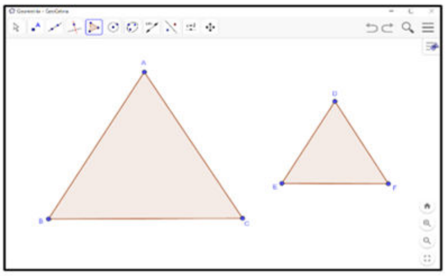
1. Using a dynamic geometry program (such as Geogebra), construct non-congruent pairs of triangles [ABC] and [DEF], such that:

1.1. the sides of the triangle [ABC] measure three times the sides of the triangle [DEF]

1.2. $B\hat{A}C = E\hat{D}F$, $\overline{ED} = \frac{\overline{BA}}{2}$ and $\overline{DF} = \frac{\overline{AC}}{2}$

1.3. $B\hat{A}C = E\hat{D}F$ and $C\hat{B}A = F\hat{E}D$

1.4. $B\hat{A}C = E\hat{D}F$, $\overline{ED} = 4\overline{BA}$ and $\overline{EF} = 4\overline{BC}$



2. On each pair of triangles drawn, if necessary, measure sides and angle amplitudes.

Determine the ratio between the lengths of the sides of each pair of triangles represented.

Are the triangles similar?

Experiment with different triangles under the same conditions.

3. Based on the results obtained in the previous question, what conditions do you suggest should be verified when one wants to recognize whether two triangles are similar?

Translated and adapted from Almeida (2022, p. 130)

Figure 2. Task on similar triangles from a 7th grade approved textbook (Translated and adapted from Almeida, 2022, p. 130)

This task challenges students to consider the relationships between the length of the sides and the angle measurements of triangles, allowing them to determine whether or not the triangles are similar. From there, the students are expected to identify the criteria that determine triangle similarity. The dynamic geometry software can be used to experiment with a variety of triangles, enabling the students to explore and formulate conjectures.

THE CASE OF MEXICO

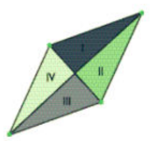
In Mexico, there have been few official government initiatives for integrating digital technologies to middle-school mathematics education. The most significant was the Teaching Mathematics with Technology programme (EMAT), launched in 1997 by the Mexican Ministry of Education (SEP) and which lasted for 11 years (see Sacristán et al., 2021). In the current (but soon to be changed) 2017 national mathematics guidelines (SEP, 2017) for middle-school, the use of digital resources are recommended "to encourage students to explore mathematical ideas and concepts, as well analyse and model problematic phenomena and situations". Among the DT tools suggested are spreadsheets, symbolic manipulators, graphing tools, and Geogebra. However, the guidelines do make it clear that this can happen if schools have the necessary resources available. Hardly anything else is stated regarding technology in the printed guidelines, but in the corresponding website (https://www.planyprogramasdestudio.sep.gob.mx/sec-ae-pensamiento-mate1.html), for middle-school there are a few examples of technology-based activities (called "ICT use"). In particular, for the topic of "Patterns, geometric figures and equivalent expressions", there are a few references to tasks from the EMAT programme (no other technology-based activities are presented). And for the topic of "Figures and geometric bodies" for all three middle-school grades, the website states that it "is suggested to work with dynamic geometry programs, such as GeoGebra, and with activities in LOGO". All these references are very interesting and show the impact that the EMAT programme had; on the other hand, it is striking that no links are given to any of the referenced resources, other than printed pages numbers from the EMAT activity books.

Congruences of triangles and quadrilaterals with dynamic geometry at middle-school

For grade 9 students, the 2017 guidelines state as learning aim that the student "Builds similar polygons. Determines and uses similarity criteria of triangles" (SEP, 2017, p. 324), and suggests (as stated above, the use of GeoGebra or LOGO). However, we couldn't find any recent activities on this topic. There are, however, two from the EMAT middle-school Dynamic Geometry activity book (Zubieta et al., 2000), that we present in Figure 3.

Triangles and quadrilaterals: Figures directly or inversely congruent

Aim: Distinguish when two figures are directly congruent or inversely congruent.



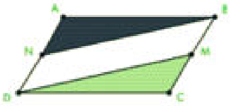
How are the triangles formed by the diagonals that cross the rhombus above?

Some are directly congruent, while others are inversely congruent.

- If the point of intersection of the diagonals is the common vertex of the four triangles, what is the value of the angle, at this common vertex, in each of the four triangles?
- Therefore, to classify triangles as directly or inversely congruent, a rotation or a reflection, respectively, will suffice.
- Which triangles are directly congruent?
- Demonstrate the above using the ROTATE command and describe what happens.
- What are the inversely congruent triangles?
- Demonstrate the above using the REFLECTION command and describe.

Triangles and quadrilateral: How to check the congruence of figures

Aim: To use translation and rotation to verify congruence between two figures.



The drawing shows the parallelogram ABCD and the midpoints of the opposite sides, namely BC and AD.

- How are triangles ABN and CDM related to each other?
- Will there be a rotation that puts one triangle on top of the other?
- What point could be the centre and what is the value of the angle?
- To place one triangle on top of another, you could also perform a translation and then a rotation (or vice versa); in this situation it would be necessary to indicate with a vector the direction of the translation.
- Could you point out that vector in the drawing?
- Then, you would have to choose a point as the centre and the value of the angle for that rotation. Which would it be?
- Try to do it with both methods and describe what happened.

Figure 3. Mexican middle-school dynamic geometry tasks on congruence of triangles and quadrilaterals (Adapted and translated from the EMAT book by Zubieta et al., 2000)

As we can see, these are guided tasks to build and explore (through reflection questions) congruent figures with Dynamic Geometry.

THE CASE OF COLOMBIA

In Colombia, in the national guidelines (MEN, 2006), the use of digital technology is recommended but not mandatory. It is recommended to use technologies to favour the solution of problems and the communication of mathematical ideas in class. Concerning resources, in the official textbooks the use of digital technologies is also recommended. For example, in the textbook for grade 8 (MEN, 2015), GeoGebra is used to study different topics (e.g., function) but its use is scarce in the entire geometry unit in general. On the other hand, the Colombian national government promotes an official teaching repository (www.colombiaaprende.edu.co) created, with the support of South Korea, for a project on ICT training. These resources include different kinds of resources for teaching with didactical suggestions and complementary activities for students: e.g., lessons planned by teachers and shared with colleagues; other resources like videos, tests and specific Apps. However, it does not include specific math-oriented software.

Interactive apps for the teaching of similarity in 8th grade

In the Colombian mathematics curriculum the teaching and learning of similarity is proposed as a fundamental teaching content in spatial thinking from the first grades of primary-school to the last grade of secondary school. In middle-school, for grades 8-9, the guidelines (MEN, 2006) state as learning aims: "I conjecture and verify properties of congruence and similarity between two-dimensional figures and between three-dimensional objects in solving problems" and it is intended that students apply and justify the use of congruence and similarity criteria between triangles. Looking for teaching resources with DT for such topic, it was noteworthy that the national official repository (https://www.colombiaaprende.edu.co/en/node/91283) does not include anything using dynamic geometry (nor for any other topic).

However, the repository does provide 8th grade interactive apps for the topic (see Figure 4). However, these could be considered as computer-aided instruction (CAI) tasks, that translate what could be solved with paper-and-pencil, rather than provide any technology-enhanced mathematical thinking.

Figure 4. An activity on similarity of figures from the Colombian national digital resources' repository.

A CONCLUDING REFLECTION ON THE EXAMPLES

Both the Italian and Colombian examples are resources for teachers that include didactical suggestions, although the Italian one highlights a complete methodology for technology integration. In contrast, Portugal and Mexico's examples are tasks for students only.

We observe that three of the examples propose explorations with DG, but with very different approaches. In the case of Mexico, it is a guided construction with questions directed towards specific properties. In the Portugal example, the approach is more open and exploratory, and the questions are not only for identifying the conditions needed for triangles to be similar, but on creating an awareness of why they are true. Finally, the case of Italy also promotes constructions and from the constructions the identification of geometric properties; but the example stands out because it includes heuristics and explicitly promotes a working methodology (including collaborative work and discussion).

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