A PROPOSAL OF MIXED METHOD ANALYSIS IN MATHEMATICS EDUCATION USING FUZZY COGNITIVE MAP

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Qualitative research is often identified as idiographic: a research focused on cases and avoided generalizations. Whereas quantitative research is identified as nomothetic by proposing the determination of general laws. Most of the time, quantitative and qualitative research should not be regarded as two paradigms, as two different worldviews are unable to communicate. In this paper, we first focus on the main characteristics of the two kinds of research analyses. Then, an approach based on Fuzzy Cognitive Maps is proposed as a mixed method analysis to be used in mathematics education to combine qualitative and quantitative research results. Finally, some examples of applications are discussed.

Keywords: qualitative research, quantitative research, Fuzzy Cognitive Map, mixed methods.

INTRODUCTION AND MOTIVATIONS

The empirical research line in Mathematics education has two major traditions: one commonly defined as quantitative, oriented towards numerical quantification with nomothetic intentions, and one commonly defined as qualitative with idiographic intentions. The nomothetic approach starts with a hypothesis and, thus, a prediction. Having studied situations, it is possible to recognize them and predict what will happen when the same circumstances recur. In the idiographic approach, an explanation of the studied episode is obtained after it has happened: we speak of a retrodiction. The two traditions have often focused on different salient features to the extent that they are perceived as opposing ontological, epistemological, and methodological levels. However, in today's research landscape, the idea is increasingly gaining ground that in quantitative research, there must be moments in which the concepts to be quantified are rigorously defined, and this requires a prior qualitative analysis of these concepts that can make use of epistemological and methodological facilities typical of qualitative approaches. Similarly, in qualitative research, there is the problem of how to generalize "unique cases," and therefore, conceptual categories on which quantification operations can be carried out must be regularly defined. In several cases, it is important to recognize that qualitative and quantitative research methods in mathematics education are not mutually exclusive. Many researchers use both approaches in their studies to gain a more comprehensive and nuanced understanding of the phenomena they are studying (Johnson et al., 2007). Rather than being seen as two separate paradigms, qualitative and quantitative research could be viewed as complementary approaches that can be used to address different aspects of a research question. Kelle and Buchholtz (2015) point to the limitations of a purely qualitative approach. They critically review the continuing dispute about qualitative and quantitative research methods that overshadow research in mathematics education. These authors question the restriction to either quantitative or qualitative methods, which they find particularly striking in research on teacher knowledge. They argue how a "mixed methods design" can enrich mathematics educational research in this domain. Hart and her colleagues (2009) examined 710 research articles on mathematics education in six prominent educational journals during the period 1995-2005, finding that 50% of the studies used qualitative methods only, 21% used quantitative methods only, and 29% mixed qualitative and quantitative methods in various ways. In this study, an approach based on the use of Fuzzy Cognitive Maps (FCM) (Kosko, 1986) as a mixed-method analysis is proposed. Starting from qualitative data, it uses an appropriate categorization of the observed elements to provide the quantitative data of an educational phenomenon. The authors have already applied this methodology in some studies in the field of mathematics education (Capone & Lepore, 2021; Capone et al., 2022) and in some studies of humanistic informatics (D'Aniello et al., 2021). Some examples of applications of FCM in Mathematics Education are shown here, highlighting how it can be generalized to other studies.

QUALITATIVE RESEARCH

Qualitative research is often used to explore the experiences and perspectives of students and teachers in specific contexts. This approach typically involves collecting data through observations, interviews, and other qualitative methods to develop a deep understanding of the phenomena being studied (Bikner et al., 2015). Qualitative research is often associated with idiographic methods, which focus on individual cases' unique characteristics and features (Cobb, 2007). In 1974, at the assembly of the American Psychological Association, Donald T. Campbell argued that qualitative knowledge "precedes" and guides quantitative knowledge and that the second depends largely on the first. In the same assembly, Lee J. Cronbach proposed reversing the priorities followed until then in experimental studies: from generalization to individual cases, exceptions, uncontrolled conditions, personal characteristics, and events that occur during the study. Qualitative research is observational-interpretative and favors the approach of textual analysis, aiming to understand the meaning of the actors' behavior and the texts produced based on detection (observation) or by them (interviews, documents). The approach may be Hermeneutic, which is based on interpretation aimed at reconstructing the meaning of the set of an actor's actions or of a text, and Phenomenological, i.e., based on the recognition of certain themes within sets of actions or within texts that identify themes or events common to different referents (uniqueness). To this line of research belongs, for example, Anne R. Teppo (2015), who introduces grounded theory as a methodology; Vollstedt (2007), that identifies different types of personal meanings and describes conditions of their emergence, which constitute significant elements of a theory of personal meaning in mathematics learning. Other scholars focused on perspectives regarding the reconstruction of social interaction and argumentation, e.g., Götz Krummheuer (2015), Christine Knipping (2008), also following Toulmin's theory. Other scholars highlighted a methodology based on the Vygotskian perspective (Radford et al. 2005). Furthermore, various qualitative research methods that have emerged in mathematics education over the past three decades are analyzed (Bikner et al., 2015). Some examples of qualitative analysis came from Tommy Dreyfus, Rina Hershkowitz, and Baruch Schwarz that presented Abstraction in Context (2020) as a theoretical-methodological approach for researching students' knowledge constructions. Moreover, Arthur Bakker and Dolly van Eerde (2015) introduce design-based research as a specific methodological approach in realistic mathematics education. Michèle Artigue (2009) considers didactical engineering in the French tradition as a case of design research.

QUANTITATIVE RESEARCH

Quantitative research in mathematics education typically involves collecting and analyzing numerical data to identify patterns and make generalizations about larger populations. This approach is often associated with nomothetic methods, which aim to identify general laws or principles that can be applied across different contexts (Shaughnessy, 2007; Watson & Callingham, 2003). The use of quantitative data analysis methods based on statistics became widespread in the early 20th century, thanks to Karl Pearson, Udny Yule, and Ronald Fisher, who defined numerous indices and procedures. The advent of the personal computer in the 1980s lowered the data processing cost, allowing anyone to process large volumes of data. In those years, the computer generated a real euphoria for the quantitative, with an often uncritical application of data collection and analysis techniques. Research based on quantitative methods stems from a realistic ontological view and generally pursues nomothetic aims, aiming at identifying relationships between factors. It uses highly formalized data collection and analysis procedures to describe a given educational reality based on the given statistical parameters (Bolondi et al., 2019). Quantitative methods usually explain a factor based on other factors (e.g., identifying the relationship between the use of computerized teaching tools and the skills acquired by students in certain subjects). The goal is to identify and detect specific factors that leads to the use of procedures that produce highly structured data (Lee et al., 2022).

MIXED METHODS

In recent years, many scholars have used mixed methods in mathematics education (e.g., Kelle & Buchholtz, 2015). Following Johnson et al. (2007) definition, "mixed methods research is the type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches (e.g., use of qualitative and quantitative viewpoints, data collection, analysis, inference techniques) for the broad purposes of breadth and depth of understanding and corroboration. […] A mixed methods study would involve mixing within a single study; a mixed method program would involve mixing within a research program, and the mixing might occur across a closely related set of studies". (Johnson et al., 2007, p. 123). The key is to synergically use qualitative and quantitative approaches at different points in the same research to find better answers to the research questions. The question then arises of how to integrate the two approaches without overlapping them, how to find ways to integrate them, and how to interpret the results in the light of a unified framework. Some strengths of mixed methods research include simultaneous generation and testing of theory, the possibility of answering a broader extent of research questions, and added value of additional knowledge for theory and practice. In contrast, a researcher has to be firm and confident in applying multiple research methods (Kelle & Buchholtz, 2015).

FUZZY COGNITIVE MAP

In this study, Fuzzy Cognitive Analysis is proposed as a methodological framework that, starting from a qualitative analysis and appropriate categorization, attempts to give a quantitative answer to an educational phenomenon through Fuzzy Cognitive Maps. A Fuzzy Cognitive Map (FCM), as introduced by Kosko, is a symbolic representation based on a fuzzy graph useful for representing causal relationships. It can symbolically describe complex systems/environments, highlighting events, processes, and states. An FCM consists of an interconnection of nodes through weighted edges: a graph node is called a concept, and an edge is called weight. The edge allows for implementing a causal relationship between two concepts (nodes), and the weight represents the strength of the influence of the relationship, described with a fuzzy linguistic term (e.g., low, high, very high, etc.).

A FCM can be formalized through a 4-tuple (N, W, A, f), where:

1. $N = \{N_1, N_2, \ldots, N_n\}$ is the set of n concepts which are represented by the nodes of the graph.
2. W: $(N_i, N_j) \rightarrow w_{i,j}$ is a function ($N x N \rightarrow [-1,1]$) which associates the weight $w_{i,j}$ to the edge between the pair of concepts (N_i , N_j).
3. A: $N_i \rightarrow A_i$ is the activation function which associates to each concept N_i a sequence of activation values, one for each time instant t: $\forall t$, $A_i(t) \in [0,1]$ is the activation value of the concept N_i at time t.

4. $A(0) \in [0,1]$ n is the initial activation vector containing the initial values of all the concepts; $A(t) \in [0,1]$ n is the state vector at a certain instant t.
5. f: $R \rightarrow [0,1]$ is a transformation function with a recursive relation $t \geq 0$ between A(t+1) and A(t):

$$\forall i \in \{1, \dots, n\}, A(t+1) = f\left(\sum_{\substack{i=1 \\ j \neq i}}^{n} w_{ji} A_j(t) \right)$$

Different functions can be used as f (x), such as the sigmoid function, the bivalent function or the linear function. FCM can be used to make a what-if inference, starting from a given initial activation vector A(0), to understand what will happen next to the modeled system/environment.

A Fuzzy Cognitive Map is developed by integrating existing experience and knowledge related to a system. This can be achieved by using a group of experts to describe the structure and behavior of the system under different conditions. With FCM, it is usually easy to find which factor needs to be changed, and, being dynamic modeling tools, the resolution of the system representation can be increased by applying further mapping. FCMs can be used for various purposes, such as underlining agents' behavior, understanding the reasons for their decisions and actions taken, and highlighting any distortions and limits in their representation of the situation (explanatory function). They can also be useful for predicting future decisions and actions (forecasting function) and for helping decision-makers reflect on a given situation's representation (reflexive function).

BUILDING A FUZZY COGNITIVE MAP

Before choosing FCMs as a tool to summarize the qualitative data of research quantitatively, an in-depth study of the scientific literature was conducted, evaluating different fuzz-type intelligence computing approaches proposed by the community to describe structures and behavior of agents and different dynamic situations. With respect to other fuzzy approaches, the use of FCM provides us with some advantages. For example, FCMs are based on causal cognitive mapping, which provides an efficient way to elicit and capture the knowledge of the experts of the domain and provide an intuitive way to represent such knowledge that can be easily managed and updated by such experts.

Typically, an FCM results from a construction process involving several experts. The approach used by the authors is presented here. The designed FCMs, proposed by the authors in their works, resulted from a consensus process in which a team of four education experts worked. Starting from one or more methodological frameworks typically used in mathematics education and considered useful to describe ongoing research, each expert has proposed a FCM to identify the causal relationships and weights between the available concepts. The weights are represented by several linguistic terms, for example they could be: no impact = 0.00, very low = 0.165, low = 0.335, medium = 0.50, almost high = 0.665, high = 0.835, very high = 1.00. Then, the experts aggregate the different maps proposed to obtain one FCM. When some differences arose between the relationships and weights proposed by the experts, they discussed these differences. They tried to find an agreement until they reached a sufficient consensus. The authors proposed FCMs can be considered organized in different layers. The lowest level usually contains the concepts that represent the atomic variables, allowing a numerical correspondence with the observed phenomenon to be clearly established. The numerical values provided as input could be made available by software data collection tools or simply by a human operator consistently assigning a numerical value based on the observed phenomenon. For example, suppose we are interested in modeling the engagement level of an undergraduate mathematics student, a low-level concept might represent the number of completed assignments. The activation levels of these “leaves” concepts represent the value of each variable. When these variables' values change, the other concepts of the FCM are influenced according to the causal relationships between them. The middle layer usually contains the nodes composing the top-level concepts of the model they are intended to represent. For example, if the high-level concept to be modelled is motivation, the intermediate-level concepts contributing to its definition could be intrinsic motivation, extrinsic motivation, and social motivation.

USING A FUZZY COGNITIVE MAP

In several research studies, we sought quantitative confirmation of what emerged from the qualitative study. For example, we created a situation model using a customized Fuzzy Cognitive Map capable of mapping the students' cognitive processes in the activities proposed during the research, referring to specific methodological frameworks of mathematics didactics. Specifically, using a suitably designed Fuzzy Cognitive Map allowed it to transform qualitative analysis into quantitative analysis and compare different approaches and situations. To provide concrete evidence of this approach, two different examples of the application of FCMs that we built in our research are given. Specifically, are presented a custom FCM to assess the impact of changing environment on undergraduate mathematics students’ status, and a custom FCM to assess whether the use of Augmented Reality (AR) can improve the conceptualization of specific mathematical objects. The two custom FCMs, apart from the different goals they are intended to achieve, differ mainly in the theoretical frameworks they use and how the numerical values of the low-level concepts are set. We start by describing the motivations that led to the building of the first map, then we look at its structure, and finally, we sum up the major findings. Mathematics teaching with STEM undergraduate students has evolved over the past three years; it has moved from blending to fully distance learning during the pandemic. We were interested in how students reacted to changing environments from a cognitive point of view. Data collected by our situation ware e-learning platform were summarized through engagement, participation, and motivation, which described students’ status, calculated through an ad hoc fuzzy cognitive map. The values of the low-level map concepts were provided directly by the implemented software platform. The FCM that is being referred to is shown in figure 1. As can be seen, the top-level concepts are engagement, motivation, and participation, and they are the roots from which all other nodes branch out to the leaves. An accurate description of concepts that make up the map can be found in (Capone & Lepore, 2021). The results of the engagement, motivation, and participation levels through the years and experiences highlighted the positive and negative aspects of using technology in mathematics teaching. Thus, as a methodological implication of this research, we proposed a teaching method (called Integrated Digital Learning) that integrates moments of distance teaching with activities carried out in the presence, classroom, or other university environments. A mix of styles, a fluid flow of knowledge between the physical classroom and the virtual classroom.

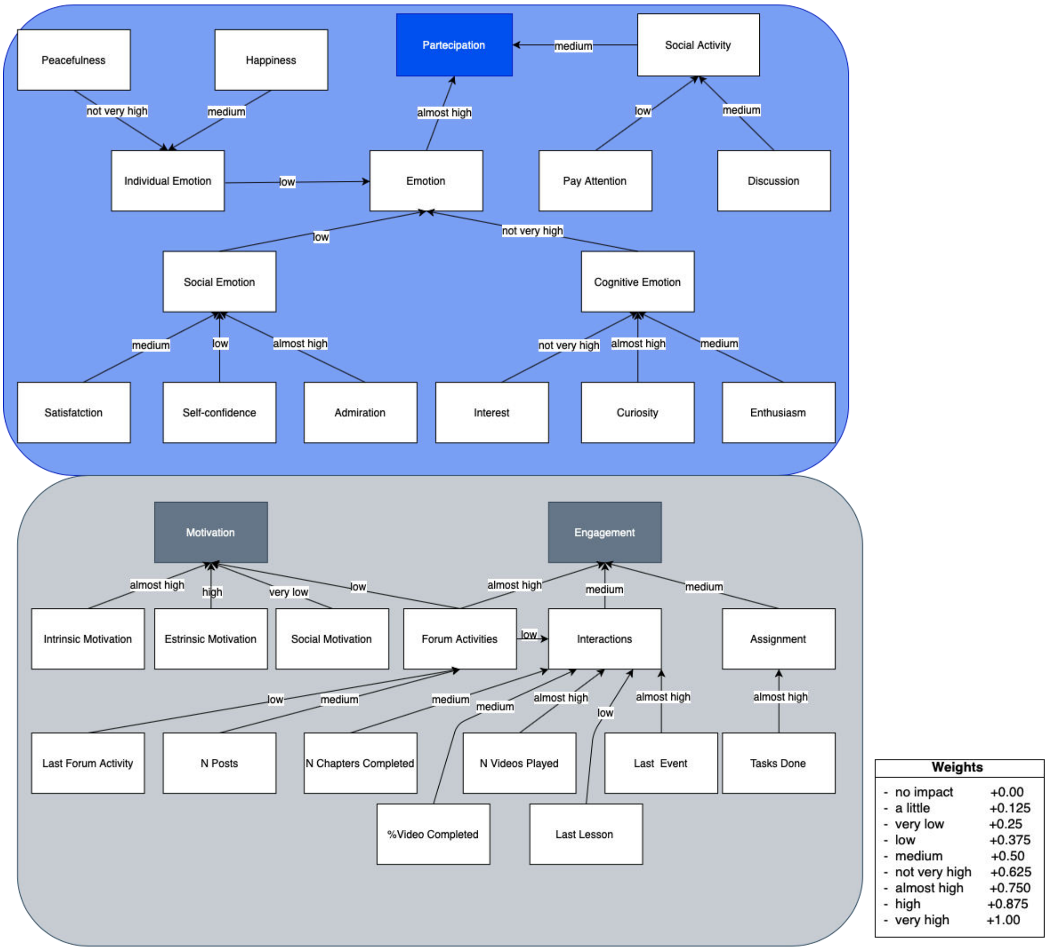

Figure 1: FCM describing student's status in terms of engagement, motivation, and participation

As a second example, an FCM to assess the utility of Augmented Reality is given. Our research focused on designing and implementing teaching strategies using GeoGebra Augmented Reality to improve conceptualizing specific mathematical objects. Referring to Duval's theory of semiotic representations, our research hypothesis is that AR facilitates students' transition between different semiotic representations, boosting their motivation and engagement and improving their mathematical conceptualization process. We describe a teaching sequence on paraboloids, framed by the Method of Varied Inquiry and Marton's Variation Theory, which involved thirty undergraduate students in Mathematics. The research methodology was not limited to a qualitative analysis of video-recorded activity. Still, it was also based on a further quantitative analysis of the learner's status, described through the top-level concepts of conceptualization, inquiry, and emotions, using a custom Fuzzy Cognitive Map we reported in Figure 2. The low layer contains *discussions with peers* and *discussion with teachers*, associated with the *top-level inquiry* concept, which help describe the amount of student interaction with peers and teachers, respectively; *intuition, visualization, representation, recognition, and encoding*, associated with the *conceptualization* top-level concept, which has been used to express the students' level of ability of treatment and conversion, according to Duval's theory; *satisfaction, self-confidence, admiration, peacefulness, happiness, interest, curiosity, and excitement*, linked to *emotion* top-level concept, which were chosen as the most important feelings to describe the student's emotional state while performing the activities. In this research, a human expert operator estimated numeric values for the low-level concepts from the available material acquired during the experimentation.

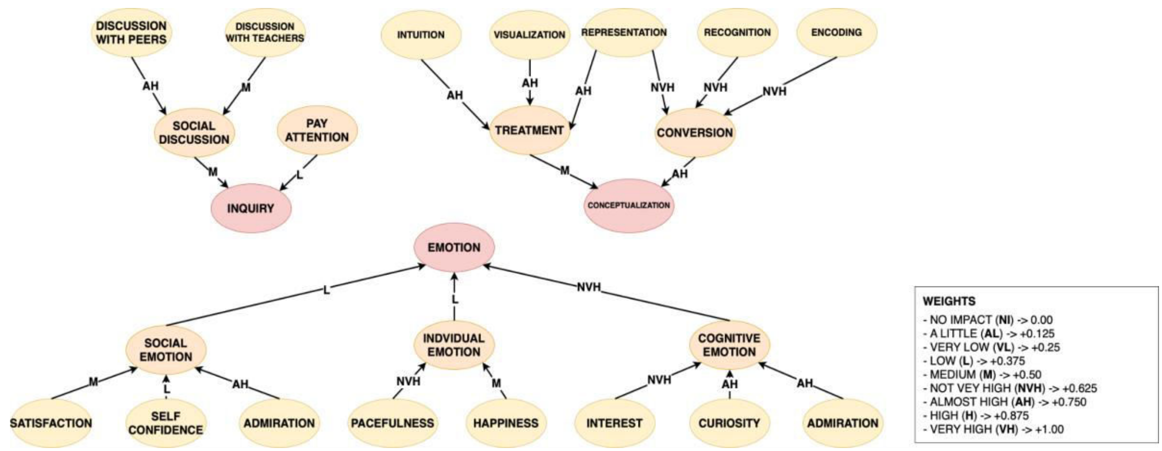

Figure 2: FCM for student's emotion, inquiry, and conceptualization evaluation

The results show how the AR activity successfully engaged and guided students to characterize paraboloids by easing the transition between semiotic representations, from the graphical to the analytical and vice versa.

CONCLUSIONS

To sum up, Fuzzy Cognitive Map (FCM) is widely used in various fields, including mathematics education. FCM is a graph used to represent knowledge and its causal relationships. It allows the visualization of the complexity of a system and the identification of the most relevant factors. In mathematics education, we used FCM to conduct a mixed-method analysis using qualitative and quantitative data. The methodology involves categorizing observed elements, which are then represented in the FCM. Some examples of the application of FCM in mathematics education include the analysis of students' problem-solving strategies, the identification of the most effective teaching methods, and the evaluation of the impact of technology on learning outcomes. The use of FCM in mathematics education is not limited to these examples and can be generalized to other educational phenomena. Overall, using FCM as a mixed method analysis in education provides a valuable tool for researchers and educators to understand better and improve educational phenomena. As future developments, we are working on defining a mathematics education framework of data analysis based on describing students' cognitive processes concerning the literature's main and most widely used frameworks.

REFERENCES

Artigue, M. (2009). Didactical design in mathematics education. In *Nordic research in mathematics education* (pp. 5-16). Brill.

Bakker, A., & Van Eerde, D. (2015). An introduction to design-based research with an example from statistics education. *Approaches to qualitative research in mathematics education: Examples of methodology and methods*, 429-466.

Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. (2015). Approaches to qualitative research in mathematics education. *USA: Norma Presmeg*.

Bolondi, G., Branchetti, L., & Giberti, C. (2018). A quantitative methodology for analyzing the impact of the formulation of a mathematical item on students learning assessment. *Studies in Educational Evaluation*, *58*, 37-50.

Capone, R., & Lepore, M. From Distance Learning to Integrated Digi-tal Learning: A Fuzzy Cognitive Analysis Focused on Engagement, Motivation, and Participation During COVID-19 Pandemic. *Technology, Knowledge and Learning*, 1-31. (2021).

Capone, R., De Falco, M., & Lepore, M. (2022b). Improving under-graduate students' performance through a Situation Aware e-learning system in *Proceedings of the Third Workshop on Technology Enhanced Learning Environments for Blended Education*, Foggia

Cobb, P. (2007). Foundations 1. Putting Philosophy to Work: Coping With Multiple Theoretical Perspectives. In F. K. Lester(, Jr Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp1-38). Reston: The National Council of Teachers of Mathematics

D'Aniello, G., Gaeta, M., Lepore, M., & Perone, M. (2021). Knowledge- driven fuzzy consensus model for team formation. Expert Systems with Applications, 184, 115522.

Hershkowitz, R., Dreyfus, T., & Schwarz, B. B. (2020). Abstraction in Context. *Encyclopedia of Mathematics Education*, 9-13.

Kelle, U., & Buchholtz, N. (2015). The combination of qualitative and quantitative research methods in mathematics education: A "mixed methods" study on the development of the professional knowledge of teachers. *Approaches to qualitative research in mathematics education: Examples of methodology and methods*, 321-361.

Knipping, C. (2008). A method for revealing structures of argumentations in classroom proving processes. *ZDM*, *40*(3), 427-441.

Kosko, B. (1986). Fuzzy cognitive maps. *International journal of man-machine studies*, *24*(1), 65-75.

Krummheuer, G. (2015). Methods for reconstructing processes of argumentation and participation in primary mathematics classroom interaction. *Approaches to qualitative research in mathematics education: Examples of methodology and methods*, 51-74.

Lee, H. Y., Chung, C. Y., & Wei, G. (2022). Research on technological pedagogical and Content Knowledge: A Bibliometric Analysis from 2011 to 2020. In Frontiers in education (Vol. 7, p. 765233).

Radford, L., Bardini, C., Sabena, C., Diallo, P., & Simbagoye, A. (2005). On Embodiment, Artifacts, and Signs: A Semiotic-Cultural Perspective on Mathematical Thinking. *International Group for the Psychology of Mathematics Education*, *4*, 113-120.

Shaughnessy, J. M.(2007). Research on statistics learning and reasoning.In F. K. Lester(, Jr Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp 957- 1009). Reston: The National Council of Teachers of Mathematics.

Teppo, A. R. (2015). Grounded theory methods. *Approaches to qualitative research in mathematics education: Examples of methodology and methods*, 3-21.

Vollstedt, M. (2007). The construction of personal meaning: A comparative case study in Hong Kong and Germany. In *European Research in Mathematics Education V. Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2473-2482).

Watson, J. M. &Callingham R. (2003). Statistical literacy: A complex hierarchical construct. Statistics Education Research Journal, 2(2), 3-46.