FEELING THE SLOPE: LEARNING THE DERIVATIVE CONCEPT WITH AUGMENTED REALITY

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The affordances of augmented reality technology allow the design of learning environments that can foster the embodied learning of calculus concepts. The augmented reality design presented in this contribution allows students to follow a function graph by hand, so as to simultaneously create the related derivative graph. This contribution explores how the students' bodily interaction with the function graph may help them become aware of the function-derivative relationship. Grounding our work in the notion of meaning-making as a process of objectification, we qualitatively analyzed the video recordings and transcript of the learning experiment involving a tenth-grader student. The results present three focuses of mathematical awareness the student achieved through the mediation of his body.

Keywords: augmented reality, function, derivative, meaning-making, objectification

INTRODUCTION

Meaning-making of mathematical concepts is one of the themes that has drawn the attention of mathematics education researchers for decades, and a well-consolidated direction of research concerns the meaning-making of calculus concepts (e.g., Thompson, 2013; Swidan et al., 2020). The idea that cognitive development in mathematical thinking grows from perceptions and actions to formal productions (Tall, 2009) and the recent focus on embodied cognition (Wilson, 2002) have motivated researchers to explore how students make meaning of calculus concepts when they use different kinds of digital tools. Dynamic and multiple representations tools were extensively used to explore how students make meaning by them (e.g., Noss et al., 1997; Swidan, 2022); embodied artifacts have been used recently to examine the meaning-making of the rate of change (Swanson & Trininc, 2021), and the augmented reality sandbox tool was adopted to help undergraduate students to make meaning of the gradient concept (Bos et al., 2022).

The affordances of augmented reality (AR) technology allow the design of digital tools that can foster the embodied learning of calculus concepts. This use of augmented reality has not been extensively explored, and its potential for advanced meaning-making of calculus concepts is still blurred. This study explores how a specific augmented reality design can help students make meaning of the derivative concept. The AR design presented here allows students to trace a function graph by hand, and the derivative graph simultaneously appears in front of the students. In particular, the aim of this contribution is to explore how the students' bodily interaction with the function graph may help them become aware of the function-derivative relationship.

THEORETICAL FRAMEWORK

In this study, meaning-making is considered to be a process of objectification (Radford, 2003). In other words, meaning-making is a matter of actively endowing with meaning the conceptual objects made available by the AR artifact used in the study. Meaning-making as objectification requires making use of different semiotic means such as words, symbols, actions with the artifact, rhythmic speech, and gestures available in the universe of the discourse (Radford, 2003). Radford referred to this process as a "semiotic means of objectification." From a pragmatic view, Radford suggests a semiotic tool to analyze educational mathematical activities in the classroom. The basic components of the semiotic tool are the *students' attention* and *awareness* of the mathematical object. A variety of semiotic means of objectification that have a representational function attract the students' attention to mathematical objects. Furthermore, the artifact's properties can help students attend to the mathematical objects related to the activity under consideration. Students become aware of the attributes of mathematical objects within that phenomenon by paying attention to the necessary aspects of the mathematical phenomenon and using various semiotic means of objectification. By being aware, students attain objectification of the mathematical objects, which then become apparent to them through various devices and signs.

METHOD

The learning experiment

The learning experiment described here is conducted in a lab condition with the support of an AR headset called Magic Leap: it is a pair of glasses through which the user can see the reality and the augmented objects. In the default interface of the application designed for the study purpose, the student can choose among seven elementary types of functions (upper part of Figure 1a). In the lower part of the interface, the gestures useful to interact with the technology are recalled jointly with their role (Figure 1a). When selecting one of the functions, a Cartesian system with numbered axes appears showing the selected function in blue color (Figure 1b). The learner is asked to move his hand along the graph: simultaneously with the hand's movement, a green tangent line appears, and the derivative curve is sketched point by point in the same color (Figure 1b). Moreover, a yellow number denoting the value of the slope of the tangent line is displayed. The student should be close enough to the function graph so that the tangent line is displayed. Otherwise, the line becomes red instead of green, and the derivative graph is not created (Figure 1c).

The design principles underlying this software were inspired by Alberto et al. (2019) work in which students can create the graph of a function by coordinating their hands' movement along a specific constraint. The a priori hypotheses behind such a learning environment are mainly two. First, since the environment juxtaposes the function graph, its derivative function graph, and the hand movement, students should come up with conjectures about the mathematical relationship between the two graphs. Second, body involvement should help the students to find that relationship by 'feeling' the slope behavior as they move and control their hands.

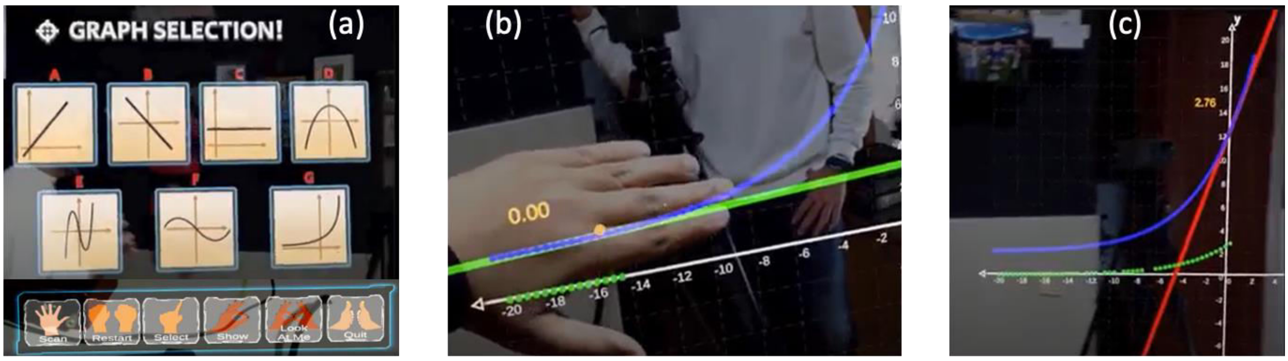

Figure 1. a) The software interface; b) A green tangent line and the derivative graph are created simultaneously with the hand's movement; c) If the student is not close enough to the graph, the tangent line becomes red

The learner and the researcher

The learner involved in this experiment is Karim, a 10th-grade Israeli student, who participated on a voluntary basis. He is a high achiever student and learns mathematics according to the high level of the Israeli curriculum. By the time of the experiment, he had already learned the linear function, the quadratic function, and reading graphs in the Cartesian system. The student was familiar with the slope concept but not yet familiar with the derivative concept. The third author, Omar Abu Asbe, who also designed the software and so knew well how to manage it, was present throughout the duration of the experiment. The task given to Karim requested him to produce some conjectures about the relationship between the blue (the function) and the green (the derivative) graph and to elaborate on the meaning of the yellow number (the slope). The researcher and Karim were engaged in the learning experiment for a full hour.

Data collection and data analysis

All the session was video recorded with a camera focused on Karim. The researcher could always see what Karim was seeing through his headset due to a camera integrated into the headset and projected on a wider screen. This data was also recorded and analyzed. The two video clips of the session, the one focused on Karim and the one showing what he could see through his headset, were synchronized so that they could be watched simultaneously. The joint video was then transcribed and translated from Arabic into English. Each segment of the video and related lines of the transcript were analyzed according to the processes of noticing and focuses of awareness as mentioned in Swidan and Fried (2021). A focus of awareness should be intended as "a node in the learning process, a point at which students' attention becomes drawn to a component of the mathematical idea via the digital technology and they begin to seek to enlarge their awareness of that component's meaning and its place in the general decomposition of the idea" (p. 5). This gradual process of becoming aware happens through a "creative process of finding or noticing something" (Radford, 2008, p. 225). Hence, the preliminary analysis here presented consisted of a two-step coding: first, describing the mathematical elements Karim noticed during the learning experiment; then, based on these elements, focuses of mathematical awareness were elaborated by identifying the mathematical knowledge related to the function-derivative relationship Karim became aware of by the mediation of the elements he noticed. At the end of the coding phase, the video clips were divided into episodes illustrating Karim's attempts to generate conjectures about the function-derivative relationship. The episodes were organized into categories based on the elements he noticed. For example, episodes concerned with Karim's meaning-making of the yellow number were grouped into the category "the objectification of the yellow number."

RESULTS

In this section, we present several short episodes showing three different focuses of awareness. Karim's sentences were often interspersed with long pauses. To make the best use of the space available, we will merge his statements using the notation [#n#] to indicate a pause of n seconds between sentences.

Focus a): The objectification of the green points

At the beginning of the learning experiment, Karim selected the increasing line (Figure 1a, option A) and moved his hand slowly along the straight blue line, which signifies the function graph (Figure 2a). Once Karim moved his hand along the function graph, green dots, which signify the derivative function graph, appeared simultaneously with his hand movement (Figure 2b and 2c). While Karim was entirely focused on his endeavor, the researcher requested him to describe what he saw.

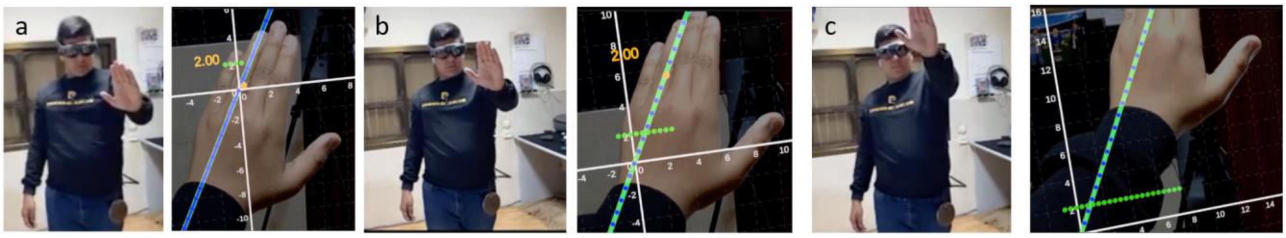

Figure 2. a) Karim moved his hand along the blue line; b) and c) His hand movement produced the appearance of the green points

1	Researcher:	Karim, go back and tell me what you see
2	Karim:	The entire (blue) function [#3#] the line is straight (gesture with his hand a straight line as the function graph, Figure 3a) [#2#] The points I was going through (on the blue line) [#4#] Each point I went through appeared [#2#] Appeared on this straight line (gesture with his hand a horizontal line, Figure 3b) [#8#] There is a number 2.00

Figure 3. a) Karim gestures with his finger a straight line as the function graph; b) Karim gestures with his hand a horizontal line formed by the green points

In this episode, Karim noticed, based on perceptual actions, that the green dots, which signify the derivative function graph, appeared due to his hand movement on the function graph. First, he noticed that the function graph is a straight line (linear function). In doing so, he used the words “entire function” simultaneously with gesturing the linear function (Figure 3a). Then Karim focused his attention on the blue points on the function graph. He used the verb “going through” to express his awareness of the blue dots on the function graph. It seems that the blue points, the perceptual action “Each point I went through,” and the green dots’ dynamicity helped Karim notice a relationship between the function graph and the derivative function graph (each point on the function graph appeared on the derivative function graph). Mathematically speaking, it seems that Karim noticed that the two functions share the same x-value.

Focus b): The objectification of the yellow number

In the following episodes, Karim raised a conjecture about the displayed yellow number, which signifies the inclination of his hand. First, he elaborated on this meaning when he explored the constant function (Figure 1a, option C) and then selected other functions to check the conjecture.

3	Karim:	The number is 0.00 [#5#] I expect this number is [#2#] The inclination of my hand (he positions his hand horizontally, Figure 4a) [#5#] The line I go along with my hand (moving his hand along the blue function, Figure 4b) [#2#] When I go along this line with my hand in this way [#6#] The number was 0000 [#3#] The inclination of my hand was constant [#6#] My hand didn’t tilt like this or that (tilting his hand, Figure 4c)

Karim noticed that the yellow number was 0 and endowed it with the meaning of the inclination of his hand, which was horizontal. Karim gestured the inclination simultaneously when he articulated the word “inclination” (Figure 4a) to connect the number and the inclination of his hand. He also connected it to “the line” he went along with his hand reproducing the same movement he did previously while he was interacting with the graph (Figure 4b). Then he expressed his awareness of the connection between the inclination and the constant graph “when I go along this line in this way with my hand” and then also referred to the yellow number “The number was 0000”. Furthermore, he explained what he meant by constant inclination with a gestural action signifying no tilting of the hand (Figure 4c). It seems that the perceptual action “the line I go along with my hand” and the way of positioning his hand helped Karim to notice a relationship between the hand inclination and the yellow number. Mathematically speaking, it seems that by the mediation of Karim’s hand, he objectified the relationship between the value of the slope and the inclination of the tangent line.

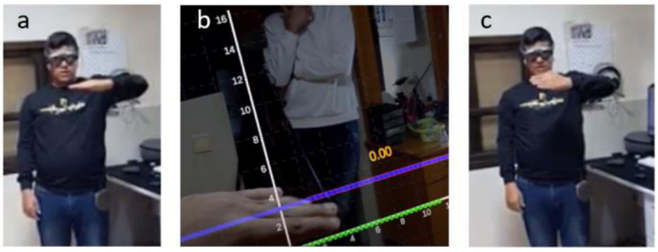

Figure 4. a) Karim positioned his hand horizontally; b) Karim moved his hand along the blue function; c) Karim tilted his hand

After the previous episode, Karim selected the increasing linear function (Figure 1a, option A) and kept on checking his conjecture.

4 Karim:	When the function is increasing (moving his hand along the function, Figure 5a)
5 Researcher:	Okay
6 Karim:	And I make my hand this way (Figure 5a) [#3#] In this movement upward on the function (with his finger he reproduces the trend of the movement, Figure 5b) [#3#] On the curve [#1#] It displays 2.00 (pointing at the number, Figure 5c) [#3#] I expect that the inclination of my hand was two (positioning his hand inclined, Figure 5d)

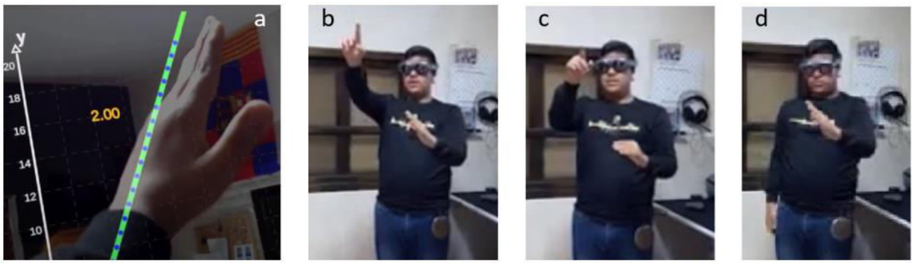

Figure 5. a) Karim moved his hand along the function; b) Karim reproduced with his finger the upward trend of the line he moved along; c) Karim pointed at the yellow number; d) Karim positioned his hand inclined

In this except, Karim selected the increasing linear function and moved along it (Figure 5a). While he was moving his hand he noticed both the inclination of his hand and the displayed number. Indeed, immediately after, he reproduced with his finger the “movement upward” of his hand

(Figure 5b), and then he pointed at the number 2 shown on the curve (Figure 5c). Eventually, he concluded that he assumed the inclination of his hand was 2 while simultaneously tilting his hand (Figure 5d). It seems that to check his conjecture, Karim noticed the same elements of the previous episode, hand inclination and number, not only in words but also through perceptual actions, hand and finger gestures. The way of positioning his hand helped Karim consolidate his mathematical awareness of the relationship between the value of the slope and the inclination of the tangent line, but as Karim himself remarked when saying “I expect”, it is still a conjecture to be validated.

In this third episode, Karim switched from working with the increasing linear function to the decreasing one (Figure 1a, option B), with the purpose of verifying the relationship between the yellow number and the inclination of his hand.

7	Karim:	Now when I go along it (Figure 6a), the number becomes -2 (pointing at the number, Figure 6b) [#12#] Because the inclination of my hand is downward in this way (Figure 6c) [#5#] I expect that this is the reason why the number is negative and 2

Also here, Karim noticed that the yellow number, which signifies the slope value, became -2 when he selected the decreasing linear graph. Karim noticed that the yellow number became negative because he tilted his hand downward in a certain way (Figure 6c). Karim also remembered that in the previous case, the number displayed was 2. Indeed, Karim connected all the elements previously noticed. He argued that “the number is negative” because “the inclination of his hand is downward” and the number is 2 because the hand is tilted “in this way”. Hence, Karim’s awareness of the relationship between the number displayed and the inclination of his hand increased: the sign (negative or positive) of the number depends on the direction of the hand (upward/downward) while the value of the number depends on the inclination.

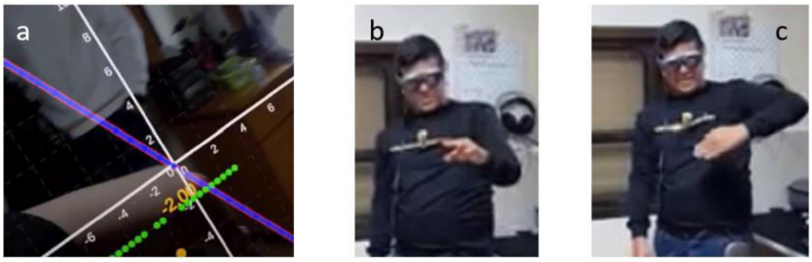

Figure 6. a) Karim moved his hand along the decreasing function; b) Karim pointed at the yellow number; c) Karim tilted his hand downward

Focus c): The objectification of the change in the slope value

Immediately after the previous episode, in which Karim explored functions that have a constant slope, he chose the cubic function graph (Figure 1a, option E). He moved his hand along the function until reaching the maximum point. Then he selected the parabola (Figure 1a, option D). He was moving along the function with his hand while he connected the yellow number and the inclination of his hand to the points on the function.

8	Karim:	Once I reach this point (pointing gesture, Figure 7a), which is straight, it becomes 0.00
		[Karim selects the parabola and moves along its graph]
9	Karim:	In this point (pointing gesture, Figure 7b) the number is 4.00 [#6#] I expect the inclination of my hand is 4.00 [#9#] As I move in this direction the number is decreasing.

10 Researcher: Why?

11 Karim: Because the inclination of my hand (inclined hand held by the other hand, Figure 7c) decreased relative to this position [#14#] Here when I start to go downward [#4#] The number becomes negative [#2#] It is -2.80 [#5#] It starts [#7#] It starts to increase [#2#] Unlike here

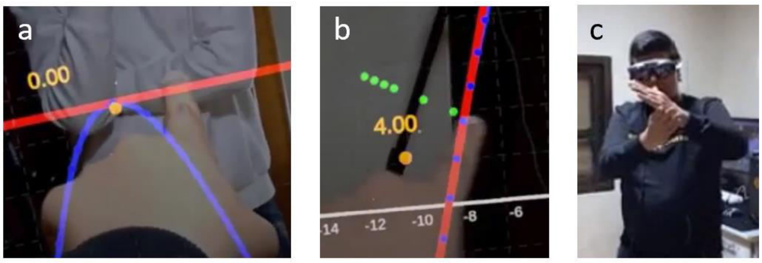

Figure 7. a) Karim pointed at the yellow number; b) Karim positioned his hand horizontally; c) Karim inclined his hand and held it with the other hand

In this episode, Karim noticed the value of the number and the inclination of his hand in relation to the maximum point of the selected function. At this moment, it seems that Karim became fully aware of the relationship between the inclination of the hand, the number, and the points on the function. When selecting the parabola, he shifted his attention from focusing on the relationship between his hand inclination and the slope value in specific points of the function toward focusing on this relationship through a domain of the function "As I move to this direction the number is decreasing." Here Karim used the verb 'move' to describe his hand movement, and used the verb 'decrease' to objectify the change in the slope values. Then, Karim connected his hand inclination with the numerical values and the direction of his hand movement by the term negative.

DISCUSSION

The preliminary data analysis described in this contribution concerns three main focuses of awareness. Focus a) shows how Karim became aware that the green points are connected to the function graph. This happened due to the hand movement along the function which produced the simultaneous creation of the green points. Throughout focus b), Karim became aware of the yellow number, and its connection to the inclination of his hand. Perhaps, the fixation of the yellow number drew his attention, or the continuous movement of his hand in the same direction. Indeed, he chose various linear functions: first constant, then increasing and decreasing. The change in the direction of his hand helped him to notice the sign of the slope endowing it with mathematical meaning (negative or positive). It seems that the freedom in tilting the student's hand in the space together with the appearance of the number that measures the hand inclination, allowed Karim to evaluate his hand inclination, which eventually helped him to objectify the relationship between his hand movement and the slope value. In focus c) Karim noticed the change in the slope values in different points of the function and then in domains of the function. Feeling the change of slope moving along the same graph helped him elaborate on his previous mathematical awareness. Further steps of analysis will address all the focuses of awareness throughout the entire learning experiment and investigate how a given focus of awareness involved the use of others.

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REFERENCES

Alberto, R., Bakker, A., Walker-van Aalst, O., Boon, P., & Drijvers, P. (2019). Networking theories in design research: An embodied instrumentation case study in trigonometry. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education* (CERME 11) (pp. 3088–3095). Utrecht University; ERME.

Bos, R., Doorman, M., Drijvers, P., & Shvarts, A. (2022). Embodied design using augmented reality: the case of the gradient. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *41*(2), 125–141.

Noss, R., Healy, L., & Hoyles, C. (1997). The construction of mathematical meanings: Connecting the visual with the symbolic. *Educational Studies in Mathematics*, *33*(2), 203–233.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning*, *5*(1), 37–70.

Radford, L. (2008). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Sxhubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture* (pp. 215–234). Sense Publishers.

Swanson, H., & Trninic, D. (2021). Stepping out of rhythm: an embodied artifact for noticing rate of change. *Educational Technology Research and Development*, *69*(4), 2037–2057.

Swidan, O. (2022). Meaning making through collective argumentation: The role of students' argumentative discourse in their exploration of the graphic relationship between a function and its anti-derivative. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *41*(2), 92–109.

Swidan, O., & Fried, M. (2021). Focuses of awareness in the process of learning the fundamental theorem of calculus with digital technologies. *The Journal of Mathematical Behavior*, 62, 100847.

Swidan, O., Sabena, C., & Arzarello, F. (2020). Disclosure of mathematical relationships with a digital tool: a three layer-model of meaning. *Educational Studies in Mathematics*, 103, 83–101.

Tall, D. O. (2009). Dynamic mathematics and the blending of knowledge structures in the calculus. *ZDM*, *41*(4), 481–492.

Thompson, P. W. (2013). In the absence of meaning... . In K. Leatham (Ed.), *Vital directions for research in mathematics education* (pp. 57-93). Springer.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic bulletin & review*, *9*(4), 625–636.