Dynamic tessellations in support of the inquiry-based learning of mathematics and arts

Toni Chehlarova, *toni.chehlarova@gmail.com*
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

Evgenia Sendova, *jenny@.math.bas.bg*
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

Elisaveta Stefanova, *sou73sof@abv.bg*
73/ General High School "Vladislav Gramatik" (Second German Language School), Sofia

Abstract

The paper presents an example of a didactic scenario illustrating the inquiry-based learning (IBL) as implemented in two educational projects 25 years apart. The message is that even when different computer environments are used (Logo and GeoGebra in this case) it is the constructionists' approach that is decisive for the learning outcomes.

Keywords

Tessellations, dynamic software, inquiry-based learning, mathematics congruencies, art

The inquiry-based learning – an aspect of the constructionism

The phrase *inquiry-based learning* is used to describe a process in which the students (usually under the guidance of a teacher) are"discovering" (possibly "creating") knowledge themselves by carrying out experiments, by formulating and investigating conjectures, by discussing and sharing with peers both ideas and products – the latter being an important component of the constructionism. Such an approach is relatively new in the context of the mathematics education and has been promoted in Bulgaria mainly since 1984 within the experimental schools of the Research Group on Education (RGE) (Sendova, 2011). The main principles behind the RGE educational philosophy were "learning by doing" and "integration of the school subjects" and they were extended beyond the RGE curriculum in the *Mathematics and Informatics* textbooks for the secondary schools (1988-1989) in Bulgaria (Sendova & Nikolov, 1989).

Problem solving scenarios in *Mathematics and Informatics* textbooks

The main questions addressed in these textbooks were: *how to feed curiosity, wonder and excitement in mathematics and informatics classes*; *what kind of projects and scenarios would keep up the students' attention for a longer period*; *how to provoke the exploration and discussions on the process of development*. Two ideas which turned out to be fruitful were:

- to include problem solving scenarios that could be developed both vertically (in grades) and horizontally (within the same grade)
- to apply the integral approach, i.e. to let the students see one and the same notion reflected in different contexts.

In one of the scenarios (appearing in the textbooks for 9th and 10th grade) the students acted as designers of wallpapers and tessellations by means of the Logo turtle geometry. The project started with design of motifs to be replicated in the nodes of a regular grid thus forming a wall-paper. When playing experimentally with different inputs of the wall-paper Logo procedure (corresponding to the type of the motif and to the distance between the gird nodes) the students could find out that the results were very sensitive even to small changes of the input values and that some new shapes might appear as a side effect. The *tessellations* – a special case of wall-papers in which the motifs interlock perfectly to fill the plane without gaps or overlapping, proved to be an object of exploration with a great appeal to the students. The tasks in the context of *tessellations* can be organised so that the students would have to combine mathematics and informatics skills of different level, e.g. to find all the regular polygons tessellating the plane, to generate the polygon-tiles simultaneously by means of multiple turtles (Blaho & Kalas, 1998), recursive procedures (Clayson, 1988), to modify the tessellating regular polygon by implementing geometric transformations such as *congruences* (translations, rotations, reflections and compositions of those) so as to obtain a tile with a new shape (Sendova & Grkovska, 2005; Chehlarova & Sendova, 2010). The latter idea can be easily implemented by means of dynamic software which we did in the frames of two European Projects dealing with the inquiry based learning (IBL) in science and mathematics.

Some recent developments of the IBL

The projects under discussion are *InnoMathEd* (Innovations in Mathematics Education on European Level) and *Fibonacci* (Disseminating inquiry-based science and mathematics education in Europe) (Kenderov, 2010; Chehlarova et. al., 2011). The Inquiry Based Mathematics Education (IBME) in the frames of these two projects has been promoted in Bulgaria at two levels – national, and local (in major regional centers) (Kenderov, Sendova, 2011).

The specifics of the teachers training courses was the variety of the audience. The school principals would often form groups of teachers from the primary and secondary school, teachers in mathematics, informatics, ICT, and sometimes even in science, arts, and history so as to gain a critical mass of teachers able to implement the IBL by means of dynamic computer environments. Thus the teacher educators had to introduce relatively new dynamic software (*GeoGebra*, *Geonext*, *Elica* applications for 3D explorations) for a couple of days in the context of didactic scenarios developed by the Bulgarian *Fibonacci* team in harmony with the curriculum (but not limited to it) and to demonstrate the inquiry-based teaching/learning process. The participating teachers experienced the potential of the learning environments specially designed (i) to support a joint work among teachers and students acting like a research team in which the teacher acts as a discovery-guide; (ii) to encourage students to find their own learning paths according to their interests and potential, and to build the knowledge in a cross-disciplinary context, especially integrating mathematics with IT, natural science and art (http://www.math.bas.bg/omi/Fibonacci/archive.htm).

What follows is an excerpt of a *Fibonacci* learning environment in the context of tessellations and its implementation in a class setting.

How to transform dynamically a square in a new tessellating tile

Let us illustrate the idea of dynamic tessellations by transforming a square tile in a tessellation tile of a new shape. We construct the square as a partial case of the *polygon* tool, select a point **E** on its side **AB** and a point **F –** on the segment **EB.** Then we construct an arbitrary point **M** and the images of **E, F** and **M** under translation by vector **AD.** Connecting the points as shown in the third picture of Figure 1 we get a newly shaped tessellation tile.

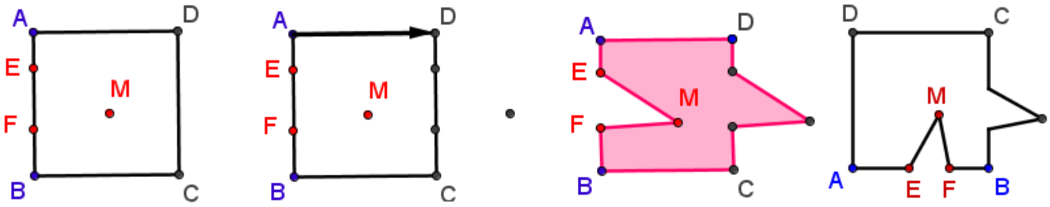

Figure 1. Transforming a square into newly shaped tiles

It is possible to transform the square in another way — let us construct now the images of **E**, **F** and **M** under rotation with center **B** and angle $-90°$. Connecting the points as shown in the fourth picture of Figure 1 we get another tessellating tile. Next we can get a module of four tiles by means of translation (Figure 2 upper left) or rotation (Figure 2 bottom left).

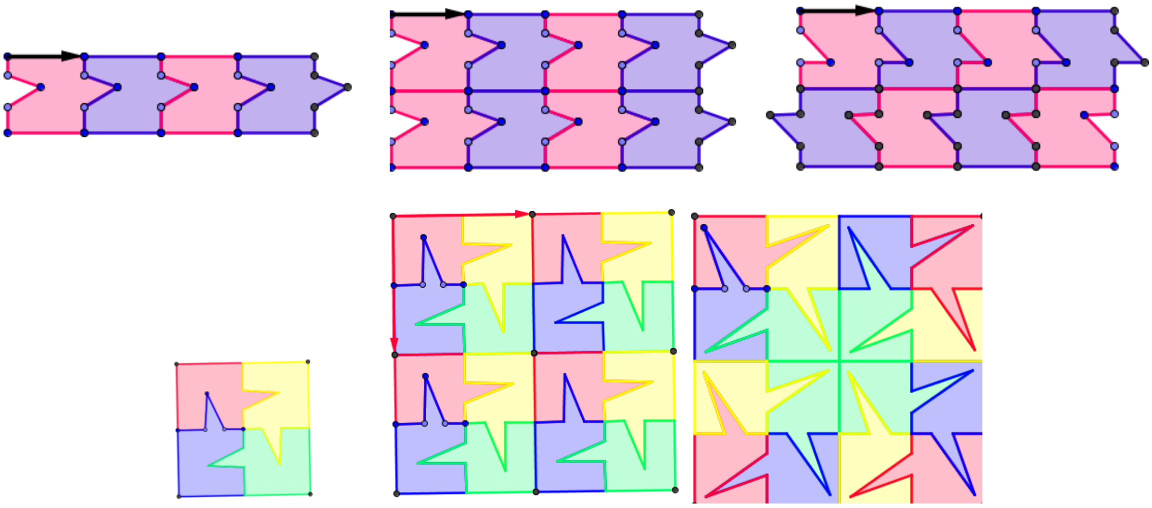

Figure 2. Constructing a module of tiles by translation and spreading it in various ways

The next steps could be carried out in various modes (by applying a translation, a central symmetry or reflection) which assures a great variety even with such a simple starting shape. Furthermore, the variety of tessellating tiles could be achieved by free movements of the points **M**, **E**, **F (**Figure 3**)**

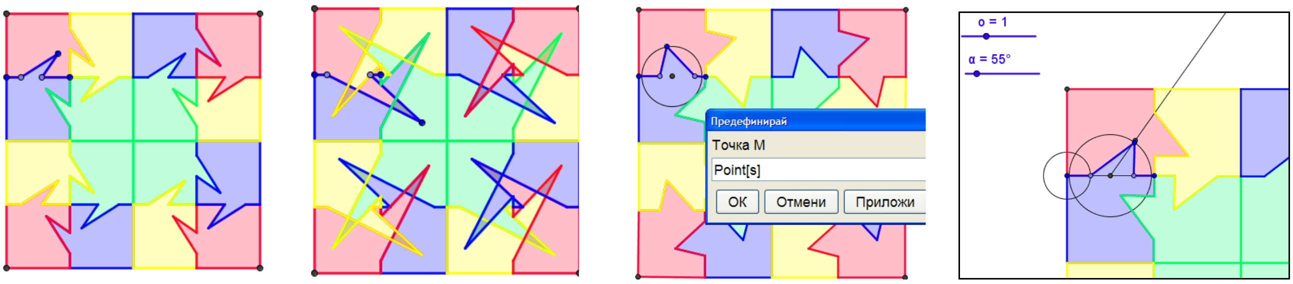

Figure 3. Variations by free movements of the points and a new construction to avoid self-intersection

Since **M** was chosen arbitrarily such a free movement would allow for self-intersection of the contour of the original tile (Figure 3 – the second picture) which contradicts to the idea of keeping its property of being a tessellating tile. To avoid this problem we shall choose point **M** on a preliminary constructed object, viz. a circle ***s*** with diameter close to (but smaller than) **AB**. We construct scrollers ***o*** and ***α*** to automise the movement of the points **E**, **F**. For the purpose we re-define point **E** as intersection point of the segment ***a*** = **AB** and a circle with center **A** and radius ***o*** (varied by the scroller ***o***). In order to be sure that such a point exists we fix **A** and **B** and assign an appropriate value for the upper boundary of ***o*** depending on the length of ***a*** = **AB**. Next we construct point **M** as intersection point of the circle ***s*** with the second leg of the angle with a vertex the midpoint of **AB**, a first leg passing through **B** and a measure in grades ***α*** (varied by the scroller ***α)*** . What is left is to hide the unnecessary elements and start the animation mode of the scrollers. These are only some of the ideas implemented in the context of the tessellation scenario but even they gave the impulse for working in exploratory style to teachers and students alike.

Playing Escher in a class setting

This scenario was presented (by the first two authors) with detailed instructions in a Bulgarian journal in mathematics and informatics. The third author (a teacher within the *Fibonacci* project) *took the gauntlet* and implemented it with 7-graders in IT classes. Here is what she shared at the bi-weekly seminar of the *Fibonacci* project: *The students started with the regular polygons tessellating the plane and followed the ideas of transforming a tile by means of dynamic constructions as presented in the scenario above. Soon they realised that they had discovered their own land for explorations – playing in the style of Escher by adding new points on the initial tessellating tile (square, triangle, hexagon, rhombus) and modifying them under various congruences so as to get beautiful tiling shapes – flowers, animals, traditional martenitsa' figures, small pieces of adornment, made of white and red yarn, and warn in March (Figure 6).*

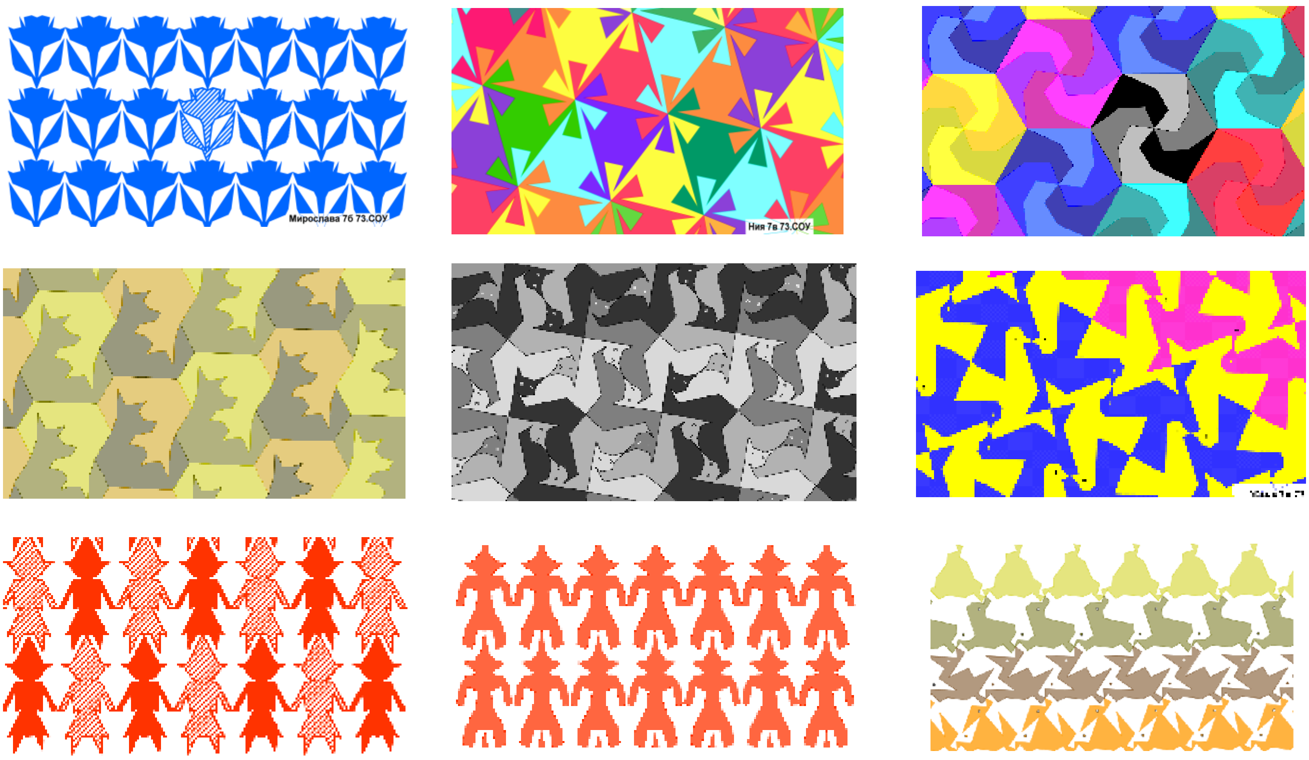

Figure 6. Students' esherizations – *flowers, animals, martenitsa's little people*

The truth is that the students discovered things that were new to me and we shared the joy. The temporary failures didn't discourage us. Sometimes we were looking for one thing and we *discovered another, or changed the direction of our explorations. It was with a great pleasure for me to realize that students who thought they didn't like mathematics all of a sudden became very active. In addition, I felt the support of colleagues the school management and the parents. As for our future plans, we already started a deeper inquiry on Escher and found that not only his tessellations but also his metamorphoses are inspirational. Here are their first attempts in this direction: (Figure 6 – bottom-right). The inquiry-based approach to learning bridged the usual generation gap between teachers and students – not only do they learn from us, but for sure we can learn a lot of new things from them and about them…*

Conclusion

The best works of the students were published on the *Fibonacc*i website and later presented in the form of book markers, greeting cards and framed paintings at a seminar within the *41st Spring Conference of the Union of Bulgarian Mathematicians* (Borovets, April 9-12, 2012). This was one more evidence that even within different computer environments the key factor for good learning outcomes is a didactic scenario tuned to the students' interests. Furthermore, we expect these outcomes to have a long-term effect since the students have worked in inquiry-based style and have *constructed a public entity*.

References

Blaho, A. & Kalas, I. (1998). *Learning by developing,* Logotron, 112-136)

Clayson, J. (1988). *Visual Modeling with Logo*. MIT Press, 15-126

Chehlarova, T. & Sendova, E. (2010). Stimulating different intelligences in a congruence context. In: *Proceedings for Constructionism 2010. The 12th EuroLogo conference*. 16-20 August, Paris, France. ISBN 978-80-89186-65-5 (Proc)

Chehlarova, T., Dimkova, D. Kenderov, P. & Sendova, E. (2011). Seeing the innovations as an opportunity, not a threat: lessons from the *InnoMathEd* European project, *Spring Conference of the Union of the Bulgarian Mathematicians and Informaticians*, April 5-9, Borovets, 347-355

Kenderov, P. (2010). Innovations in mathematics education: European projects *InnoMathEd* and *Fibonacci*. *Proceedings of the 39th Spring Conference of the UBM*, Albena, Bulgaria, 63-72

Kenderov, P. & Sendova, E. (2011). Enhancing the inquiry based mathematics education, UNESCO International Workshop : *Re-designing Institutional Policies and Practices to Enhance the Quality of Teaching through Innovative Use of Digital Technologies*, 14 – 16 June., Sofia, 56-70

Sendov, B. & Dicheva, D. (1988). A Mathematical Laboratory in Logo Style, in Lovis, F. and Tagg E.D. (eds.), Computers in Education – *Proceedings of the IFIP TC3 European Conference on Computers in Education (ECCE'88)*, Lausanne, Switzerland, North Holland, 213-223

Sendova, E. (2011). Assisting the art of discovery at school age – a Bulgarian experience http://worldtalent.ruseleon.com/libros/Bulgaria.pdf

Sendova, E. & Grkovska, S. (2005) Vsisual Modelling as a Motivation for Studying Mathematics and Art, EUROLOGO'2 005). In *Gregorczyk, G et al (Eds.) Proceedings, EUROLOGO'2005* , 12-23

Sendova, E. & Nikolov, R. (1989). Problem Solving Scenarios in Secondary School Textbooks in Informatics and Mathematics. In G. Schuyten & M. Valcke (Eds.) *Proceedings of the Second European Logo Conference*, Gent, Belgium, 685-693.