# Representational systems on 3d navigation process

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## Abstract

*Twenty four 10th Grade students participated in a constructivist teaching experiment, the aim of which was to engage children with concepts related to the two systems of reference used to navigate in 3d space, geographical and spherical coordinates, as well as with the relationship between them. The result showed the utilization of the new representations provided by dynamic digital media such as Cruislet provide a challenging learning context where different mathematical concepts and abilities are embedded and interconnected. Moreover the half-baked microworld approach in activity design, seemed to engaged students in the process of instrumentation and instrumentalization by exploiting the rules of the provided game and then by setting their own rules resulting on the development of new games.*

## Keywords

*navigation, mathematics, geographical coordinates, spherical coordinates, half-baked games*

A number of research studies focused on the design of exploratory media based on the principle of integrated and interdependent mathematical representations. Kynigos (2001) introduce the term of half-baked microworlds defining the microwords that are designed for instrumentalization through constructionist activity, i.e. they incorporate an interesting idea but at the same time invite changes to their functionalities and are mediated to the targeted users as unfinished artefacts which need their input. With respect to mathematical content, the approach is to identify conceptual fields (Vergnaud, 1991) which with the use of this kind of media become rich in the potential to generate mathematical meanings, irrespective of the ways in which they might be structured (or fragmented) in the mathematics curricula.

In the design of such kind of microworlds, the principle of providing students with mathematizing activities seems to be effective. Keisoglou and Kynigos (2006), focusing on the meanings that students formed during the mathematization of a science-like measurement activity point out the potential role of the students' involvement in problem situations that are experientially real. The idea of mathematization activity involve the students' engagment in a particular real problem within the context of a microworld designed for instrumentalization. Experimenting, constructing classifications, making and verifying conjectures , generalisations and formalizations are a number of activities that should lead to mathematisation.

The ‘Cruislet’ environment is a state-of-the-art dynamic digital artefact that has been designed and developed within the ReMath project. It is a digital medium based on GIS (Geographic Information Systems) technology that incorporates a Logo programming language. Cruislet was conceived as a digital medium for mathematically driven navigations in virtual 3-d geographical spaces. Users can view avatar positions and define their displacements by employing either a

Cartesian lat-long-height system or a vector-differential (φ,θ,ρ) system where ρ is the length of the vector of displacement. The Logo programmability is considered necessary as it provides users with the option to actually anticipate the result of their action and engage in expression of mathematical ideas through meaningful formalism by means of programming. In this sense, Cruislet can be conceived as a constructionist medium (Kafai & Resnick, 1996) in that the user can construct flights and build dependency between flights.

A digital medium (an instrument) is internalised collaboratively by the students (Mariotti, 2002) while it is being changed often quite distinctly to what was designed by the researchers. Relatively, the implication of this perspective is that students' expressions can gain mathematical legitimacy, even if they differ from and/or they are shaped and structured by the artifact in ways that lead them to diverge from curriculum mathematics. This kind of constructionist environments provides dynamic visual means that support immediate visualization of multiple linked representations (Kaput, 1992). The key point here is that students can build their models into the medium that can act as a support for developing new meanings by investigating their hypothesis and argumentation in social contexts.

Our approach to learning promotes investigation through the design of activities that offer a research framework to investigate purposeful ways that allow children to appreciate the utility of mathematical ideas (Ainley & Pratt, 2002). In this context, our approach is to design tasks for either exclusively mathematical activities or multi-domain projects containing a mathematical element within the theme which can be considered as marginalized or obscure within the official mathematics curriculum (Yiannoutsou & Kynigos, 2004).

We adopted the approach of students' gradual mathematization within game-like activities in problem situations that are experientially relevant to students. Hence, our intention was to involve students in activities through which they would use symbols, make and verify hypotheses in order to solve a particular real problem in a rich collaborative learning environment. Within the framework of instrumental genesis, we particularly focus on instrumentalization, i.e. the ways in which students learn through making changes to the digital artefact at hand. We studied the idea of pedagogical design of artefacts so that students would inevitably poke, tweak and make changes to their functionalities as part of their mathematizations. Consequently, we saw a helpful relevance in studying mathematizations in a constructionist environment as path towards clarifying the idea of instrumentalization by design.

The focus of this study is on the kind of choices do students make between spherical and geographical coordinate systems while navigating in geographical space. Specifically, we will try to investigate how students realise the role of the different representational systems on 3d navigation process and what kind of relationships they build between them.

# The Cruislet environment

Cruislet constitutes a new digital medium within the context of more than a decade of ETL R&D work on designing constructionist exploratory media based on the principle of integrated and interdependent mathematical representations. The constructionist environments designed at ETL provide dynamic visual means that support immediate visualization of multiple linked representations (i.e. any action carried on a specific representation provides immediate change and feedback in all representations, Kaput, 1992). In such settings learners are engaged in constructing public entities (constructions) implying an explicit appreciation of the relationships between mathematical objects within any situation (i.e. a mathematical model of the situation). In the case of Cruislet, learner constructions are avatar trips as well as the rules of displacement. The mathematics are those underlying the use of analytic and/or vector-differential geometry, including functions, co-variation and rate of change. However, these mathematics are integrated with geo-spatial representations and information, providing opportunities for processes of mathematisation of geographical space.

A key feature of the approach of ETL is to design artifacts afforded with integrated representations. As an example, in the last decade ETL has been involved in the design of E-slate, an educational authoring system with which many different microworlds have been developed for mathematics and science. These microworlds can be characterized as hybrids between symbolic programming (such as Logo-based Turtle Geometry), dynamic manipulation (such as Dynamic Geometry Environments), simulations, information handling and geographical systems.

In designing Cruislet we wanted to integrate programming, mathematical and geographical concepts, relations and representations (figure 1).

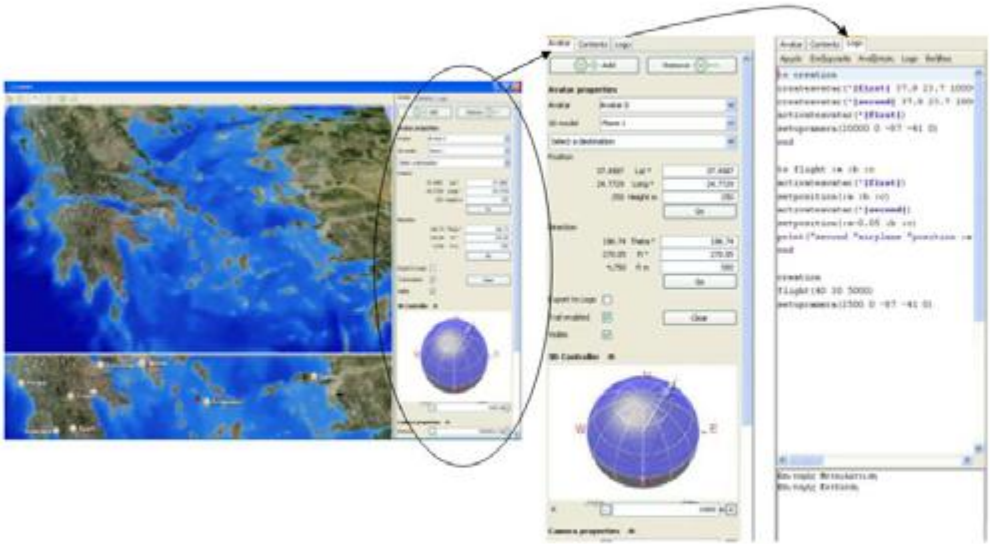

*Figure 1: Cruislet environment – Avatar Tab – Logo Tab*

New representations enabled by digital media can place spatial visualization concepts in a central role for both controlling and measuring the behaviours of objects and entities in virtual 3d environments. We have chosen the notion of vector as a means to represent the link between 2d and 3d representations, since vectors can be considered as basic components underpinning the study of geometry and motion in space facilitating the study of 3d spatial thinking. In Cruislet, a vector-differential geometrical system co-exists with a Cartesian-geographical one in an inter-dependent way. Our perspective is centered on the utilization of the different representations and the feedback that they can provide so as to facilitate multiple didactical decisions within open-ended exploratory tasks.

Moreover, navigations in virtual 3d geographical spaces within Cruislet could be conceived as game play simulations. There is a growing interest about the ways in which game–based learning environments facilitate new ways of learning (Gee, 2003). The key feature of this approach is that games can provide a context for the development of valuable skills (Kirriemuir and McFarlane et al., 2004) in the transitional stage between intuitions (informal) and formal mathematics. Using games with an appropriate set of tasks and pedagogy, students can be engaged in exploration, problem solving, rule-based thinking and other forms of mathematical thinking (Goldstein et al, 2001; Mor et al, 2004). From this point of view, the process of building game play activities involving navigation within the 3d representational space can be seen as the design of the terrain within which instrumentation/instrumentalisation processes may take place by student's interactions with the microworld and the mathematical concepts and rules embedded in it.

# Methodology

Twenty four students of the 1st grade of upper high school, (aged 15-16 years old) participated in this experiment. Students worked in pairs in the PC lab. Each pair of students worked on the tasks using Cruislet software.

The students were not accustomed in using computers for doing mathematics, but they were familiar with computers and liked using them, as almost the whole class participated in the computer class (available as a course to choose at this school level). On the other hand, concerning the concepts of geographical and spherical coordinates, none of the students had previous knowledge or experience with spherical coordinates and only four of them believed that the acquired experiences during the geography course supported their understanding of the concept of geographical coordinates. Some of the students were familiar with the basic Logo commands (movement of the turtle, such as front, right, etc.) but none of them was experienced in using programming languages. Finally, few students were familiar with map computational environments and especially with Google Earth. Nevertheless, almost all of the students were used to play computer games and most of them were familiar with 3D game environments.

Concerning the mathematical concepts that are embedded in the number of tasks in which students have been engaged, there is a considerable distance from the traditional structure of the mathematics curriculum. In a traditional mathematics class students study the concepts of Cartesian, geographical and spherical coordinate systems within abstract mathematical contexts in a rather static way. They are introduced to the concept of function through static representations provided in their textbooks without having the opportunity to manipulating or change them. Additionally, students are introduced and study the concept of vectors. (Markopoulos, Kynigos, Alexopoulou and Koukiou, 2009a;2009b)

## Tasks

The tasks are based on the idea of the "Guess my function" game, in order to provoke children to discuss, compare and experiment with dependence relations such as linear functions. Emphasis has been given to build game play activities involving navigation within the 3d representational space giving distance from the traditional structure of the mathematics curriculum. The intention was to involve students in activities through which they would use symbols, make and verify hypotheses in order to solve a particular real problem in a rich learning environment.

In the tasks that were included in the teaching experiment students were encouraged to experiment with the displacements of the two airplanes by varying the geographical coordinates of their new positions. Reflecting on their actions they encouraged to explore the rate of change of these positions and formulate the function that defines this dependent relationship. This function was hidden and the students had to guess it in the first phase of the activity based on repeated moves of airplane A and observations of the relative positions and moves of airplanes A and B.

The focus is on the functional relationships between two airplanes' relative displacements. ETL researchers consider navigation as a dynamic function event. The function's independent variable is the geographical coordinates of the position of the first airplane, which students are asked to navigate, while the dependent variable is the geographical coordinates of the position of the second airplane. We consider that the exploitation of the provided linked representations (spherical and geographical coordinates), as well as the functionalities of navigating in real 3d large scale spaces could enable students to explore and build mathematical meanings of the concept of function within a meaningful context.

The data consists of audio and screen recordings as well as students' activity sheets and notes. The data was analyzed verbatim in relation to students' interaction with the environment. In our analysis, we focused on students' actions within the provided representational contexts (visual, graphical, Logo programming) and systems (geographical and spherical coordinate systems). Students reflecting on these actions expressed their ideas, construct and developed mathematical meanings. We focused on those episodes that students seemed to realise the role of the different representational systems on 3d navigation process and built relationships between them.

# Results

Students' interaction with Cruislet environment engaged them with concepts related to the two systems of reference used to navigate in 3d space, geographical and spherical coordinates, as well as with the relationship between them. We endeavor to explore students' choices while using the two systems of reference and the ways these are manipulated in order to navigate in geographical space. Our analysis is based upon students' interaction with the available representations and their preference on one system vis-à-vis the other, while carrying out the tasks activities.

Although the case for students was to choose among coordinate systems, there were several times that they didn't choose one of them, but rather they tried to create links between the systems of reference, to navigate the airplane

## Choice according to the way of navigating.

Regarding the way of navigation, students preferred to use geographical coordinates to specify a specific position, e.g. a city on the map, in contrast to spherical coordinates used by students to make displacements in space, independently of the destination place, such as figural formations in the air. This was observed in almost all teams, despite the fact that some of them had a strong preference to one system of reference and used it to displace the airplane. In the following episode the teacher asks the class if the 3D controller (the 3D representation of spherical coordinates) is better in any case. Most of the students support this statement in a debate about systems of reference. In the thick of the conversation a student declares that this depends on the situation. The episode is interesting as it depicts students' way of thinking when they had to choose among the available systems of reference.

*R: Is Controller better in any case?*

*S1: Unless we want to go somewhere specific, for instance, at an airport. We won't use 3d controller.*

*R: Why don't we use the 3D controller to go to an airport?*

*S1: Because we have to go to the specific airport. If we go with3D controller, we'll go where it lands and we'll crash.*

*R: Nice. And how do we go to the airport?*

*S1: We insert its coordinates and it goes. (Meaning geographical coordinates)*

A similar situation occurred while another team was trying to displace the airplane in a specific position. In this case students believed that it's difficult to manipulate the airplane with spherical coordinates and that it's 'faster' to use geographical coordinates instead.

*S1: The airplane goes faster with position.*

*S2: Why? We didn't go with the other so as to know.*

*S1: Yes, but imagine. If we control it with them, we won't be able.*

The episode is interesting for another reason as well, as S1 uses the word 'control' to clarify his view of spherical coordinates. This statement is indicative of students' approach, as they viewed spherical coordinates as a way to 'control' the airplane, in contrast to geographical coordinates that displace the airplane in specific places. From our point of view, we interpret this way of viewing systems of reference as an egocentric and an absolute frame of reference, as spherical coordinates has to do with the former and geographical with the later one. As a student pointed out "*The other (meaning geographical coordinates) drives you to an area. I don't believe is as much reliable as direction, because (with direction) you can do changes on your own. Insert values, change meters you want to displace or change the degrees. Anything*.". A more detailed approach is given by the students supporting their preference in spherical coordinates." *Theta and Fi is easier, because we displace the object wherever direction we want and whatever meters we want*."

As a result of students' approach of systems of reference, they used spherical coordinates when they created figural formations in the air, although this was not included in tasks activities. An interesting example is that of a team that decided to draw letters in the air using the 3D controller representation. The figure 2 shows this construction.

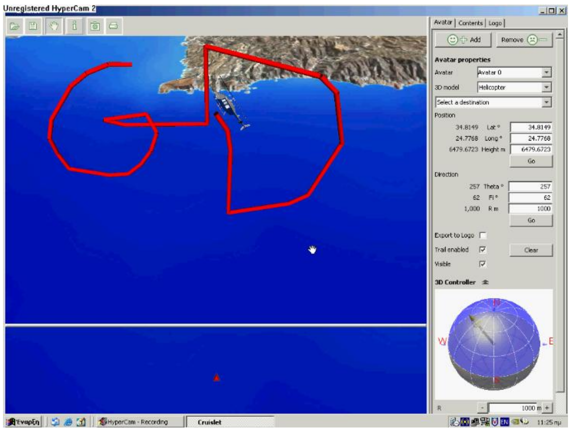

*Figure 2: Construction using the 3D controller*

## Choice among coordinates.

In this session of analysis, we report students' choices regarding the three coordinates each system composed of and how they were manipulated in order to displace the airplane.

An interesting issue is that students confronted latitude and longitude in a different way as they manipulated height in order to specify a position in space. In particular, most of the times they edited lat and long coordinates up until the airplane was displaced to a specific point of the map and afterwards they were editing the third coordinate, the height. In fact, at their experimentation, many students forgot to edit height as they were concentrated in trying to find latitude and longitude of a place. We could say that may be this is explained by the fact that they were not familiar with the environment and thought that the environment 'reminds' previous positions or coordinates. But this is not the case as such confusion occurred only with height and not with other coordinates, even if one of them remained stable. A possible interpretation about this confusion is that students are accustomed to 2d representations where they manipulate only two magnitudes and this is the reason why they usually preferred to fly at a fixed height. On the other hand if we accept the view of Dalgarno et al. (2002) that we understand 3D models through multiple 2D representations, maybe students had focused subconsciously on a simplified 2D way of visualising the displacements of the airplanes. We have to mention that although students ignored height several times, it was height coordinate that was firstly understood.

Our findings in relation to spherical coordinates are compatible with these on geographical, in the sense that students discriminated Theta and Fi coordinates from R and additionally to the fact that they were accustomed better to the late one. This was not surprising as the measures of these coordinates are different and students identified easily what each one represented. Comparing the manipulation of these spherical to geographical coordinates, we found that in this case, students also focused on changing 2 of the three coordinates (theta, fi) in order to find the right direction. Only afterwards were they editing R, that is the extent of airplane's displacement. In fact, changes in R occurred mostly when students had already made a displacement and from the result displayed in the screen, they could estimate its magnitude easier. We could say that the utilization of R independently of the other coordinates, may rely upon the fact tha they used 3D controller representation most of the times that doesn't have the R coordinate built in.

Students didn't always choose one system of reference to navigate in space, but several times combined both to make a displacement. In this way they created links either between distributed coordinates (e.g. height of geographical and fi of spherical) or between all three of coordinates for the two systems of reference.

## Links between distributed coordinates.

In their attempt to place the plane at a specific height, students used primiraly the height coordinate. However, there were some teams that were using spherical coordinates to carry out almost all displacements. Based on students actions on a team like that, students were trying to find a way to raise the airplane's height to a specific value, while utilizing the spherical coordinates. In fact one of them gave the idea to use the fi coordinate and raise the airplane by asking the other one: 'The height is fi?' and afterwards he edited the fi coordinate's value in order to raise the airplane. This statement is interesting as the student endeavor to create meaning around the fi angle that represents airplane's perpendicular angle, in relation to the height that the airplane will be placed.

Another episode where students create a link between coordinates is that of longitude and theta coordinates. In the following episode the students of a team argue about the system of refernece that displace the airplane 'right – left'.

*S2: It goes right and left. (referring to longitude)*

*E: Right and left.*

*S2: Yes.*

*S1: No. Theta is right and left.*

*S2: These are the degrees.*

*S1: Yes, the degrees it turns to the left or right.*

*S2: I'm saying to displace at the same time.*

The episode is interesting as it depicts the way students verbally express the way they realize the displacement while using longitude or theta angle of spherical coordinates. In both cases they use the expression 'right – left' giving the displacement a sense of direction. However, S2 supports that longitude doesn't have to do only with turning like theta, but with displacing as well. The way he externalizes his thought demonstrates that he is aware of the interdependent relationship between longitude and theta.

## Links between all three coordinates

The manipulation of 3D controller acted as vehicle with which students realised the notion of vector as the displacement and associated airplanes' displacement with the variation in geographical coordinates. In this way, students explored vectors' properties as they constructed links between geographical coordinates (the variables of the vector of displacement) and the spherical coordinates. In the following episode we can see how the controller is used to identify the dependent relationship between coordinates' values. In particular, the student is using the arrow to prove the way values of geographical coordinates change relatively to the arrow movement.

*R: You're saying that coordinates change. (meaning geographical coordinates)*

*S1: Yes*

*R: Increase or decrease? What happens?*

*S1: It depends on where the arrow's direction is. (moving the arrow of the 3D controller)*

Another example of controller's utilization to create links between different coordinates, is shown in the following sequence of students' interaction with the environment, where they utilize both spherical and geographical coordinates to specify a position in space.

The sequence of students' actions indicates that they endeavour to associate the displacement in 3d space through the use of both systems of reference. Initially they use the 3d controller representation (spherical coordinates) and in this way they specify a point on the map as the geographical coordinates change simultaneously. Their second action includes the setting of one of the geographical coordinates as they want to place the airplane at a specific height on the map. In this case students utilised both Cruislet functionalities and the representations provided, as they attempted to combine the two systems of reference to displace the airplane.

An interesting dialogue that demonstrates the use of the 3D controller representation as a way of combining coordinates is the following one.

*R: Why it's better? (meaning the controller)*

*S: Because it combines both.*

*R: Which?*

*S: Because it has, west, north and east and all these, we can do position. And because of the arrow, we can do theta and fi. In other words…*

*R: You confused me.*

*S: We can do position because of the North, South, West, East. And with the arrow, we can also do inclination.*

In this dialogue S endeavor to support his statement that the 3D controller is the best representation to use. In his attempt to prove this, he is trying to correlate issues regarding both systems of reference, such as geographical directions that are represented on the sphere of the controller, with the arrow that defines the direction of the intended displacement.

Concluding, we could say that in the language of Didactical Functionalities, students' choices among the different coordinates' systems were based upon the modalities of use of the available representations built in the Cruislet.

# Conclusions

Cruislet microworld is designed to provide students for instrumentalization through constructionist activity in the context of half-baked microworlds (Kynigos, 2007). In particular we use the idea of half – baked games. These are games that incorporate an interesting game idea, but they are incomplete by design in order to poke students to finish or change their rules. Thus students explored the Guess my flight game play, changed it and thus adopted both roles of player and designer of the game. From this point of view the work and play with Cruislet is based on the idea of instrumentation and instrumentalization (Guin &Trouche, 1999) since displacement rules questioned and re-defined by the students resulting in a variety of artefacts. In our analysis we focused on those incidents during the teaching experiment where students seemed to be engaged in the process of instrumentation and instrumentalization by exploiting the rules of the Guess my flight game and then by setting their own rules resulting on the development of new games.

The key point here is that students can build their models into the medium that can act as a support for developing new meanings by investigating their hypothesis and argumentation in social contexts. Displacing avatars and articulating rules of and relationships between the displacements can thus provide an action/notation context which can be a new resource for activity and construction of meanings, not so dependent on the medium for its expression. Noss and Hoyles (1996) introduced the notion of situated abstraction to describe how learners construct mathematical ideas by drawing on the linguistic and conceptual resources available for expressing them in a particular computational setting which, in turn, shapes the ways the ideas are expressed. Yet, from a social constructivist perspective, psychological and social aspects of learning can never be considered separately and the term situated abstraction captures also the synergy between them: student’s activity within a community (Lave & Wenger, 1991) both shapes and is shaped by their interaction with the available tools and those around them.

From a constructionist’s point of view, the functionalities of the new digital media such as Cruislet provide a challenging learning context where the different mathematical concepts and mathematical abilities are embedded and interconnected. The role of the built in Logo environment is crusial as it provides opportunities to students to express ideas in meaningful ways and in this way it can be seen as a medium in the transitional stage between intuitions and meaningful formalism. In the case of Cruislet, navigational mathematics becomes the core of the mathematical concepts that involves the geographical and spherical coordinate system interconnected with the visualization ability.

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