The necessity of the tangent

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Abstract

In this paper we present a didactic proposal for a scenario in teaching trigonometric numbers and particularly the tangent, designed for grade 8 students. The scenario is based on the theoretical framework of constructionism. Looking back in the history of mathematics to use those elements that led to the discovery of the concept we want to teach, we create a problem-based learning environment supported by the E-Slate software, from which we use the Turtleworld microworld. Students are engaged in a process where they have to think a way which will allow them to construct scaffolds of different lengths, but with the same slope, which will serve to build all the levels, one by one, of a pyramid.

Keywords

Pyramid, scaffold, ladder, slope, tangent, angle, constructionism

Introduction

Focusing on the constructivist ideas of Piaget and their expansion, due to Papert's theory about constructionism, one can realise the huge change that they bring to instruction and the designing of teaching. Knowledge is structured not only with the individual's experience, but especially with his active involvement in its construction, using experimentation methods, modelling and participatory notification of new cognitive acquisitions. As referred to by Kafai & Resnick, (1996) it is the actual process of learning and teaching that is compatible with constructionism.

In *Mindstorms*, back in 1980, Papert advocated "the construction of educationally powerful computational environments that will provide alternatives to traditional classrooms and traditional instruction." The same time he noted technology of that time was limited regarding its capabilities and ease of use. Since then, considerable work has been done ranging from Logo, Mindstorms, Scratch, ToonTalk etc., that incorporate a constructionist approach to learning (Girvan, C., Tangney, B. and Savage, T., 2010).

Several studies have focused on the implementation of tools as a means of mediation to provide strong visual intuitions supporting production of algebraic meanings and bridging the gap between the act and expression.

The History of Mathematics, on the other hand, provides a significant range of examples with which they can engage students to build from scratch a notion, as when appeared for the first time the necessity of its creation.

With the help of technology not only as a tool but as well as an instrument and a mediator, on one and a historical event related to the use of tangent in Ancient Egypt on the other, the students in this paper will reconstruct the concept of tangent.

Theoretical Framework

As pointed out by E. Ackermann in a bibliographic article on the differences between Piaget's and Papert's theories (2001), Psychologists and pedagogues like Piaget, Papert but also Dewey, Freynet, Freire and others from the open school movement can give us insights into: 1. How to rethink education, 2-imagine new environments, and 3- put new tools, media, and technologies at the service of the growing child. They remind us that learning, especially today, is much less about acquiring information or submitting to other people's ideas or values, than it is about putting one's own words to the world, or finding one's own voice, and exchanging our ideas with others.

Traditional teaching has received numerous criticisms for its results. Contemporary teaching methods nowadays have left aside the immediate information. It is in general accepted that students do not merely take the information provided to them, but translate it according to their own criteria based on their previous knowledge and experience. According to Piaget students have serious grounds not to discard their views thanks to an externally induced anxiety.

Also, the environment, as clearly studied and indicated by Vygotsky, where the concept environment we mean all the characteristics of local culture, such as language, instruments, people, plays its dominant role in shaping the views.

Papert clarifies that constructionism —the N word as opposed to the V word— shares the same views on learning, namely it must be *"building knowledge structures"* and this will be accomplished by progressive internalization of actions. This internalization is achieved by a very cheerful way for the students, when done in a context that allows the conscious engagement with a construction that can obtain public entity. He extends Vygotsky's views in contemporary situations, suggesting as mediation tools the digital media and computers technology.

Our teaching proposal

A few words about the rationale of our proposal

No one is to oppose that planning an activity that motivates-engages students in experiments, computations and assumptions with the aim to highlight a concept or a theorem only positive results can bring, in general.

We estimate, in addition, if such an engagement is based on the origins of the concept we want to teach, then more active participation of the students is achieved. For such scenarios, when used in the teaching of mathematics (as well as in physics), apart from the fact that the students's interest is raised, they also convey, without the need of a modern translation, the ideas and knowledge that gave birth to them, since they can approach them by experimenting, constructing and expressing in their own way.

In terms of putting it in to action, we know that Ancient Egyptians could estimate the slope of a line or a plane. In fact, they used a ratio called skd (pronounced seyket) that corresponds to our current contagent (Bunt, Jones, Bedient, 1981). We use this idea, with the appropriate didactical transformation, to lead the students in a similar situation.

The approach followed in our teaching is that of problem-based learning.

The implementation of the proposal

Our original scenario was designed to be implemented on grade 8 school students in four (4) class hours. The first two hours comprised the concept of the tangent angle as a necessary tool for the indirect calculation of an angle, whereas the following two hours using that ratio to calculate sides of a triangle similar to the original right triangle. Due to space considerations in this paper we will present only the first two class hours. The students work in groups of two or three in the computer lab, using the E-slate software, on worksheets and workbooks.

We briefly present the problem to our students and which substantially raises the following question: “While we have built the first row of blocks of the pyramid and then by means of a scaffold build the second one, where must we place the second scaffold to fill the third row? To what must we pay attention?” This can be an initial discussion on the conditions and the constraints we have in the construction, since the labors-workers will have to work in the same conditions to perform the same, ie maintain the same slope in all the scaffolds (or ladders as depicted on Figure1).
So, we have a figure as the following one, with the corresponding reflection.

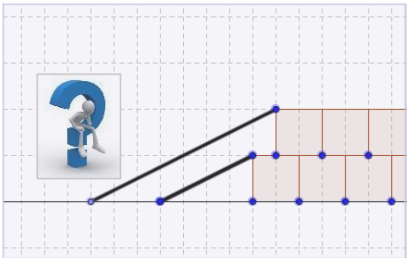

Figure 1. Where should the ladder be placed in order to construct the 3rd row?

Our aim is solely the discovery of the tangent angle. For that, we focus on the first right triangle of figure 1 omitting the remaining elements of the problem and ask students to construct this triangle. We proceed using the microworld Turtleworld of software E-slate where we present a half –baked notation code to the students, which is as follows:

```
to right_triangle :x :y :a
rt 90
fd :x
lt 180-:a
fd sqrt(power :x 2 + power :y 2)
lt 90+:a
rt :y
lt 180
end
```

And we ask to: 1. Interpret the notation code 2. To use the notation code to construct a right triangle of sides 40 and 30 turtle steps with an angle of 30 degrees. 3. Similarly, to construct a right triangle of sides 160 and 120 turtle steps with an angle as the previous one. 4. To activate the sliders of parameters (:x), (:y) and (:a), experiment and construct the two triangles.

We ask our students to interpret the notation code, to assure that all grasp it fully. Otherwise it is possible the subsequent commands that will be asked to create just to play the role of an automation button, slightly different to the ones used in their computer.

It is expected that some students will have difficulty in perceiving mentally and describing the turn of the given angle in the notation code. At this point we will explain to the students the reason why we chose to deal with the given angle. We will need to draw a figure on the blackboard, bring a parallel line from vertex C, to talk about corresponding angles etc, as follows.

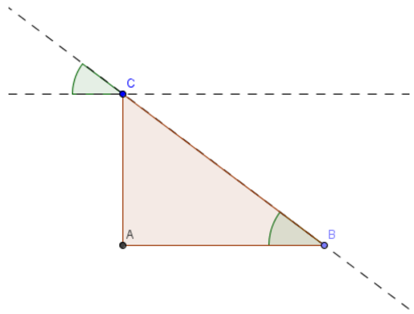

Figure 2. Corresponding angles

We ask from our students to construct two triangles, although they are meant to fail in both, for two reasons. The first triangle is too small and it doesn't provide a satisfactory visual effect, whereas the second one will reveal the weakness of the construction. Also, the sides of this triangle (the numbers corresponding to their length) in combination with the former's sides may give a first suspicion of ratios to the students.

Some groups of students while experimenting with the sliders of parameters (:x) , (:y) and (:a) -if not all- are expected to come up with an angle of 37^o that apparently leads to true results. At this point a software of dynamic geometry would be useful, to easily construct the vertical sides of the triangle and to realize, as much precision though they use, that the angle is not 37^o.

Returning to microworld *Turtleworld* and our initial construction while we ask our students to construct a triangle of sides 40 and 30 turtle steps we explain to them that the software has the ability to calculate the angle precisely, if we give a numeric value as input to the arctan ( ) command. (It would be useful to write that command in another language different to the native language of the students, if there is such a possibility from the program in order to minimize any kind of connotations that can possibly appear). We encourage our students to experiment with the sides of the triangle and the various relationships created between them.

The students will complete this phase by verifying their conjectures for the construction of the second triangle and citing their conclusions in the original problem of the construction of the pyramid.

As an epilogue

This proposal has not been implemented in class, in order to be able to give some results. The idea of the design resulted from the application of the original "historical" problem with dynamic geometry software and the results, cancelled much of our effort to lead the students to an indirect calculation of the angle. That is because the proposed constructions (similar to the ones presented in this paper) were almost all of them the result of simple applications due to the available construction tools.

For that reason when re-designing it we preferred the software *Turtleworld* due to its intrinsic geometry and continuity in design, characteristics "forcing" the students to engage with the problem in a way that leaves no room for escape in other paths.

References

Ackermann, E. (2001) "Constructivisme et Constructionisme: Quelle Différence" In *Constructivisms: Usages et Perspectives en Education* (Volume 1 et 2.) Geneva: SRED/Cahier 8. pp. 85-94

Bunt, L., Jones, P., Bedient, J. (1981) *The Historical Roots of Elementary Mathematics*, Athens, G.A. Pnevmatikos (In Greek)

Kafai Y, Resnick M. (1996). *Constructionism In Practice: Designing, thinkimg and learning in a digital world*, NJ Lawrence Erlbaum Associates, Inc.

Girvan, C., Tangney, B. and Savage, T., (2010) SLurtle Soup: a conceptual mash up of constructionist ideas and virtual worlds, *12th Eurologo vonference, Constructionism 2010*, Paris, France

Papert, S. A. (1980) *Mindstorms. Children, Computers and Powerful Ideas*, New York: Basic Books