Constructionism and the confirmation of a reluctant constructivist*

or

Why, in American mathematics education, talk of constructivism is "out," talk of constructionism never really happened, and it might not really matter anyway

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I come to bury myths, not to praise them.

Preamble

There's real physics, theology, mathematics, psychology, biomedicine, and the like, and then there's the pop-culturization of these disciplines. Our culture finds a certain appeal in (the pop-culture image of) chaos, the uncertainty principle, Freudian slips, black-holes, hormones, and so on, and uses the "explanatory power" of these ideas without much concern for the depth or context of their original meaning. People use "grew exponentially" just to mean "got really big" and "Freudian" just to mean sexy. Piagetian terms like "conservation" and "stage" are invoked in education quite casually mixed with "reinforce" and "reward" from Behaviorism.

It's easy to dismiss pop-culture as just low-brow, but I think that what's really at play is a natural adoption of ideas from one culture by another because they are useful, and the subsequent adaptation of those ideas to suit the new culture and new use. That happens even in science. Freud was studying new phenomena in a new way, and invented some completely new terms for his discoveries, but also needed a broader language and set of ideas to explain these phenomena. He certainly never believed that emotions, electric charges, and pumps were the *same*, but found the ideas from fluid dynamics and electricity *useful* enough to adopt and adapt the terms. Psychology today similarly draws many of its images and terms from computers but *adapts* them. In the case of pop-culture, it is useful to have a term for startlingly great growth, and *not* useful to worry about whether that growth is or is not really what a mathematician would call exponential. And it's culturally useful to be able to wink at "meaningful" slips and have a word to call them by without having had to study the entire field of psychoanalysis.

In American education literature, mathematics may be the earliest and most visible discusser of

* I'd thought regularly about the central event of this essay—an event involving my second grade students in late 1968—for a full twenty-five years before I could write it up. Then, awakening in a jetlagged daze at the home of Richard Noss and Celia Hoyles, I wrote the first draft of this essay in its entirety, all in one morning. Thirteen years later, that version was published in Rosamond, F. A. and L. Copes, eds. Educational Transformations: Changing our lives through mathematics; A tribute to Stephen Ira Brown. Bloomington, IN: AuthorHouse. 2006. Nearly two decades have passed since the original writing. Those second graders are now 50 year olds and I'm still learning from them. This significantly updated essay reflects that learning. Thank you, "kids."

constructivism.[1] Unlike some of the other terms, "constructivism" (much less "constructionism," which I'll get to at the very end of this essay) has not become a pop-culture word in the broader world. Too boring. In the smaller world of education, its rather specific meaning was pop-culturized to the point that it seemed to be a kind of religious or political persuasion. It is not. Despite the title of this essay—which, like any title, was selected to intrigue more than edify—it makes little sense to regard people as constructivists in the sense that they might be Italians or Masons; it is equally silly to ask if people "believe in" constructivism just as it would be silly to ask if people "believe in" quantum theory. Quantum theory and constructivism are theories, and accepting or not accepting these theories is (or should be) a matter of reason, not faith.

Neither does constructivism dictate how one teaches. Though I find no alternative to accepting the theory of constructivism, I use all the tools a reasonably flexible teacher might use (within the limits of my skills): hands-on play, lecture, demonstration, exploration and discovery, listening and responding to kids' theories—everything from sage-on-the-stage to guide-on-the-side, whatever my best (but fallible) judgment tells me might meet the needs I think my students have at a particular moment.

In the classroom, I find myself more clinician than theorist, making spur-of-the moment decisions that are more art than science. But I also find a use for theories of how people learn. They help me *think* about my teaching when I have the time and luxury to do so. Constructivism is one of those theories. This essay is neither a sales pitch for constructivism, nor a critique of it—just a clarification. In fact, these days, the word, itself, is "out," at least in the U.S., too much of a red flag. Theorize as you wish, but don't ask and don't tell. So, let's just understand the theory of learning, and forget the "ism."

Why reluctant?

Life would be very much simpler for me—as teacher, and, even more so, as curriculum writer—if I truly believed that I could place my ideas in your brain. But I don't believe I can do that. The only one with access to your brain is you. Worse yet, I don't even believe *you* can put my ideas in your brain! The only ideas you can put there are your own—ones you build yourself out of the raw material around you (including, of course, what you make of me and my ideas). For a person dedicated to helping people learn, this leaves me with far less control than I'd like. A miserable state of affairs!

So how did I come to hold such a damnably inconvenient theory about learning? I did study Piaget, and loved it, but that's not what gave me that theory-that-needs-no-name. Looking back on it now, the evidence throughout my teaching career has been so overwhelming that I could not reasonably have maintained any *other* position. Even so, one story has always stood out, certainly for its poignancy, and also as the *coup de grâce* that confirmed me as a (reluctant) holder-of-that-theory. The events of this particular story have nothing to do with mathematics, except for the irrelevant detail that I happened to have been helping Jessica with some arithmetic when the climax hit. But I'm getting ahead of myself.

[1] The word was still pretty new to us in the early 1990s, but the idea was familiar and well understood in Europe long before we got hold of it. As with most fads, the fervor is now over (and hopefully, so are the exaggeration and misuse of the underlying ideas), but the *real* ideas behind this theory remain and are still worth understanding and considering.

Swear words on the wall

It was late 1968 or perhaps early 1969, a time when riots were fresh in memory and when assassination was beginning to feel like a movement. It was a time of social fervor in many sectors: in education with the new mathematics, in social structure with feminism and Black Power, in politics and policy with the anti-war movement. In Chicago, where I was then teaching second grade, we had just gone through the traumatic summer of the Democratic convention.

Children always do things for their egos, and the story I'm about to tell might as easily have happened at any other time as at this one, but the three children involved in *this* story dropped many hints that their prank was to be seen in the larger social context. Three boys—Andy, Clark, and Mark—had a little "club." Their thing was to scream *F***** at the top of their lungs, in unison, at odd intervals throughout the day. Not too often. Just when the spirit moved them.

They would also sometimes leave the inscription in foot-high letters on the blackboard when we left the room, if nobody noticed in time. And, perhaps worst, they tormented the music teacher by doing their unison yell regularly in her class. The spirit *always* moved them when they were in music.

Consistent with the times, my assistant teacher and I were Very Understanding. Not that we didn't want to stop the disruptive behavior, but, in the spirit of that era's version of political correctness, we didn't want simply to stamp on the children's faces. So we tried all sorts of silly things, like telling the three boys that they could say whatever words they liked *to each other*, but that what they were doing involved *others*, in ways the others didn't like. As if they didn't know that! What possible fun could it be to whisper F*** to each other?!

At some point, my assistant Liz had another idea, one that I would have thought just great but didn't find out about until later when I was helping Jessica with that arithmetic. Liz thought to ask the boys if they knew what their favorite word meant. They didn't. So she explained. But she added some sociology at the same time, apparently in a style that went down quite well with the seven-year old boys.

I never found out how Liz worded this for the children, but she managed to explain to them that, though The Word was slang—a vulgar slang, at that—it had a very normal and fine meaning: it was how babies were made. She gave them the technical non-slang term using two Big Words. And the sociological perspective that she added was about people's embarrassment about "personal" things like how babies are made, and their consequent tendency to avoid talking about them, or to find substitute words (the slang), and to put all the embarrassment onto the words and thus deem the words, themselves, as *Bad*.

Clark

It was Clark who came to me to confirm Liz's story while I was sitting with Jessica. He came with what might seem a perfectly innocent question—"Is it true what Ms. K said about F***?"—but to understand the import of his question, you must know a little about Clark. His singular feature in this class was that he sucked his thumb *all the time*. Even when he spoke, he would sometimes do it through his thumb. Despite the social capital that constant thumb-sucking inevitably costs a seven-year old, he was quite popular in class, a thoroughly great kid—friendly, athletic, participating, smart, and full of lively and interesting ideas.

As it is with all children, self-image was very important to Clark, but in many ways he made it clear that self-image was a particularly special and highly personalized issue for him. He would

publicly—and always pleasantly—announce the ways he found of elevating his image. Black Is Beautiful, he'd say, but then he'd be sure that I acknowledged that this included *him*. (He *was* beautiful!) Or he'd go around the class and count the other black children. (This more than once got him flattened by a girl—the largest and strongest child in my class—who wanted her identity to be determined by *her* and not by *him*. She was quite undecided about which of her parents should play the greater role and so insisted she was neither Black nor White, but Tan!) Or he'd joke with me that he was stronger than I was: after all, he could give me a black eye, but I could not give him a white eye! And so on.

Sitting with Jessica

Anyway, one morning, as I was sitting with Jessica discussing some arithmetic, Clark came over and said, thumb in mouth, "Is it true what Ms. K said about F***?" He said just The Word, four letters unadorned. No "ing."

Despite the thumb, and the exceedingly low volume, I was quite sure I heard right. Jessica's expression showed she had heard clearly, too. I collected my thoughts and figured I just had to tough this one out, so I asked "What did Ms. K say?"

Clark then recounted his version of Liz's explanation, so remarkably ungarbled (either by a seven-year-old's version of sociology or by the thumb in his mouth) that I knew exactly what Liz had said. I was quite impressed at how well he explained this complicated matter to me, and my expression showed it. With hardly more than an "uh-huh," I confirmed that he'd got it exactly right.

His expression was of total awe. He took his thumb out—it was more like he let it drop out as both hands hung by his side—and he asked in a very serious tone "Why would God make me come from F***?! F*** is *bad!*" His exact words.

I have no memory at all of what I said.

Reluctant, yes, but confirmed

Liz's story seemed *so* hard to swallow, too hard for Clark to accept without checking it out. At the same time, neither did Clark reject the story without checking it out, because he trusted me and Liz. When I supported her story, he just had to go with it. But, what did he do? It was not *our* idea that he put in his head, but *his* idea. Our idea, right or wrong, was that people's feelings were the problem: Screaming this word, because of the feelings it aroused, was disruptive. Otherwise, The Word was like any other word—just a word—and neither it, nor what it referred to, was bad. But Clark already knew better. His idea—which, of course, seemed to him to be confirmed by our attempts to stop him and his friends—was that The Word was, indeed, Bad. *Everybody* knew *that*. We could not unteach what he knew for a fact. All we could do—if he trusted us to be right—was add a new fact, a new piece to his puzzle. He would be in total control of how that piece got used, and what additional pieces he would create in order to fit "ours" in. Because this wonderful child was working out special concerns about his own status, it comes as no surprise (with the aid of hindsight!) that he personalized the definition: "Why would God make *me* come from F***?!"

I don't remember my words, but I remember my thoughts well. I thought about Clark's pain, and what, if anything, to do, or say, or avoid doing or saying, to help (if possible) to *un*do the piece of ego damage that Liz and I had been unwitting partners with Clark in perpetrating.

But some of my thinking was pure awe at the power people have over what they hear, see, and understand. On the one hand, Liz had managed to be so clear that the complex details of her message succeeded in making their way into a seven-year-old's brain. This is a marvel of communication.[2] We are rarely so lucky! On the other hand, a young child—one who was even willing to listen to us and accept a story he found painful—managed to stand his ground against two adults and preserve his own picture of the world that we were trying to change. One had to respect Clark. He certainly *was* strong!

Well, there *is* a minor consolation in constructivist thinking. The hurt—serious and not to be ignored, but probably no more deep or permanent than many Clark had already suffered—was really a creation of Clark's, unlike many insults that are *intended* as insults. Our crime, if there was one, was not The Intent To Hurt, but something more like Reckless Messing With Someone Else's Ideas. But what else is teaching about?! Giving a definition, even for a loaded word, even with a piece of psychology/sociology, doesn't seem outside a teacher's mandate. The alternatives—behavior modification, pleading, threatening to call his mother, and half a dozen other possibilities—are equally susceptible to the kind of interpretation that our silly approach took.

Constructivism does not remove responsibility

I am acutely aware of the possibility (in fact, inevitability) that someone can interpret what I am *now* saying in a way that I do not intend. Here is the misinterpretation that I'm most worried about: When I say that we are in control only of what we do, not of how it is interpreted, I do not absolve us of the responsibility to think about how things are *likely* to be interpreted. As responsible teachers (or neighbors, parents, citizens, and so on), we must, of course, try to anticipate the responses—feelings or actions—that our words or actions may arouse. We cannot excuse insensitivity as "just words," and dismiss the consequences by observing that feelings and interpretations are, after all, constructed by the listener, and not our fault.

But the bottom line is that what goes into *your* head is what *you build yourself* and put there. I can, by experience, improve my chances of helping you build what I want you to; I can provide you richer building materials; and, perhaps by augmenting my words with pictures or manipulatives or other experiences, I can provide enough redundancy of information for you to find, somewhere, clear building instructions. But I cannot build the idea for you, nor can I put it in your head, nor can I guarantee what you will build.

This is messy!

The idea of constructivist learning is at odds with a theme that pervades education: the tendency, very likely born of desire for some control, to pre-digest information, package it in small pills (preferably sugar-coated), expect students to swallow without chewing, and expect them to digest it and incorporate it into their bodies (minds) without changing its form. The analogy fails, even for a pill. A pill cannot become part of you without changing, and even the way it changes is quite dependent on your personal chemical makeup. That's one reason you should not take someone else's pills!

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2 In fact, this particular kind of marvel could well be used to argue against "constructivism." It is a perfect example of "lecture" working, even with a young child. There were no manipulatives, he did not discover for himself, and the ideas that made their way into Clark's head did, in fact, include the ones that Liz wanted there.

The constructivist theory of learning acknowledges this messiness. Students, *all* people, construct their knowledge. We don't just hear; we interpret what we hear, and that's why people can hear the same thing differently. We don't just see; we interpret what we see, and that's why people can see the same thing differently. And we construct all our knowledge at all times: just as surely when we sit—engaged or bored—in rows in lecture classrooms as when we perform experiments with computers or manipulatives, and just as surely when we are pained, scared, and angry as when we are happy and confident. The constructivist theory is not, therefore, to be trivialized in statements like "discovery is the *best* way for students to learn." This theory says that there is no choice at all: Self-construction is the *only* way that people learn. Nor should constructivism be construed to mean that people must discover everything and be told as little as possible. It is frankly a wonder to me that *anyone* believes things like that. We *all* know better.

What does this have to do with mathematics teaching?

So what use *is* this theory? Why should we care whether teachers have their students listen to lectures and do drills or whether their students perform investigations with computers and other manipulatives and then talk about their investigations? If kids are just as surely constructing their ideas when they are sitting in lectures or slogging through a textbook, what difference does it make? What does this theory say for a mathematics teacher?[3]

For one thing, mathematics is about sense-making and logic. Many mathematical ideas are developed by children without any intervention from parents or teachers.[4] For a classic example—the one that all teachers know even if they know nothing else about Piaget's work—three-year-olds are generally convinced that taller glasses, even if they are skinny, contain more than shorter ones. If they like what is being served, they cannot be argued out of the taller glass, even by showing them—for example, by pouring from one glass to another—that their beloved tall glass actually contains *less*. And, if they don't like what's being served, they will scream for the shorter glass. Even if *we* know it's more, *they* "know" it's less. Nobody taught them that

[3] I must make another confession. Whenever I talk about "theoretical frameworks" for something as messy as teaching and learning, I feel a bit uncomfortable. Like any abstraction, a theoretical framework must simplify, must ignore parts of the data in order to be truly useful. That's much like the phenomenon I discussed in the second paragraph of this essay! In education, oversimplification is almost unavoidable. When I teach, theory can help me focus, think about the complex events of the classroom, and organize the jumble of facts into a coherent story about learning. When I'm being a theoretician, I try to build more theory, or find or do other research to clarify the story, or perhaps modify or even reject this story for a better one. Unlike a mathematical theorem, whose truth rests on logic alone and is absolute—a truth that needs no connections with a physical world—scientific theory is essentially a story whose truth lies entirely in its usefulness in explaining the events we experience and lets us to predict new events in order to make effective use of our reality. Educational theory is therefore a tricky thing. As a teacher, theory does guide me some, but not completely. I often find myself doing things that don't fully accord with what I believe, not just because I'm human and can't always act in accord with theory, but because sometimes the situation frankly doesn't seem to fit the theory and yet I must act anyway. In such cases, real science would deem the theory inadequate—it failed to account for the events—but clinical practice (teaching and psychotherapy being two good examples) requires considerable art and craft-skill along with scientific principles. It makes no sense to reject educational theories just because they don't accord with all the data. They can't accord with all the data. As a result, education lives in a fuzzy world: depending on which data we care to ignore, we generate competing theories or, worse yet, we wind up with loose or inconsistent standards for judging even the theory we've chosen to accept. Perhaps I am damaging my own credibility as a theoretician by making such a claim, but it seems the only responsible claim to make.

[4] See http://thinkmath.edc.org/index.php/Early_algebra "The Algebra of Little Kids," (Goldenberg, Mark, and Cuoco, 2010) for an analysis of some of the algebraic ideas students develop on their own before they have the arithmetic that these ideas generalize.

knowledge! They invented it themselves. Over the course of a few years, their logic changes to what we'd call the adult perspective, and they become strongly resistant to arguments that, only a few years earlier, they clung to tenaciously. This is a major mathematical step, and there are many others that children take spontaneously as they grow older.

The little child's resistance to what seems like logic to us is the same as the older child's resistance to illogic. Both are trying to make sense out of what they experience, and their unwillingness to keel over and simply accept contrary arguments is a very valuable thing. The three-year-old's conclusion about the tall glass is wrong—not all answers are right—but the reason we must respect rather than trample on the child's thinking is that we *want* people to think for themselves. It might be convenient for us in the short run if children really *did* exchange their own best thinking for our set of answers, but it would be quite unfortunate in the long run.

So, one thing that constructivist thinking tells us is that we don't want to divorce mathematics learning from sense-making. When mathematical advancement requires children to discard certain conceptions and replace them with others—and this certainly happens—we must not insult the *process* that got the children to their original, inadequate conceptions by simply declaring the results of their thinking invalid and asking them to substitute those results with ideas that *we* supply. That is like what Liz and I tried to do with Clark. It doesn't work. All that tends to happen is that the "right idea"—if it takes at all—sits beside the wrong one as an add-on, a piece of mandated *illogic*. Illogic? Yes, because if the student's logical system is not yet capable of *producing* this "right idea," then accepting it is an illogical act, an example of *un*critical thinking.[5]

Instead, we can try to find a meaningful-enough situation in which the child's current way of reasoning leads to a result that the child's own logic does not accept. The child then has an unsettling dilemma, but the child's thinking is not insulted; on the contrary, it gets credit for having recognized the conflict, and it is employed (rather than laid off) in the process of resolving the conflict. Depending on the circumstances, we might even be able to intervene respectfully in ways that truly help the child use his or her own best reasoning to resolve the conflict.

Are telling and explaining *always* bad? Of course not. When students' logical systems *are* capable of producing the "right idea," then, as long as the experience of reasoning things out for themselves is not removed from them too much of the time, there is no harm in going for the efficiency of an elegant telling. I'd still like to see students' deductive systems get a good workout a fair amount of the time—not because it's the "best way to learn (some *other*) mathematics," but because building the stamina and style to puzzle things through is, *itself*, a piece of the mathematics I think students need to learn.[6] Even this latter goal might involve "telling" students things. To the extent that this goal is served best by upping the ante of what students apply their developing logical powers to, we may want to get them efficiently (but still

5 This is not the same as saying "If the student's logical system has not yet produced this 'right idea' (but could have), then accepting it is an illogical act." If a new idea is consistent with one's own logical abilities, then accepting the idea from an outsider is not an abandonment of critical thinking: Students do not need to invent everything. But when students' own logic could not invent the new idea, we should tread lightly about asking them to accept it. A little of that experience is probably no more damaging than Clark's experience with us—an insult, but we all suffer insults and are generally pretty resilient. Too much of that kind of insult, however, gets people to give up on their own thinking, which is bad although, alas, not so uncommon.

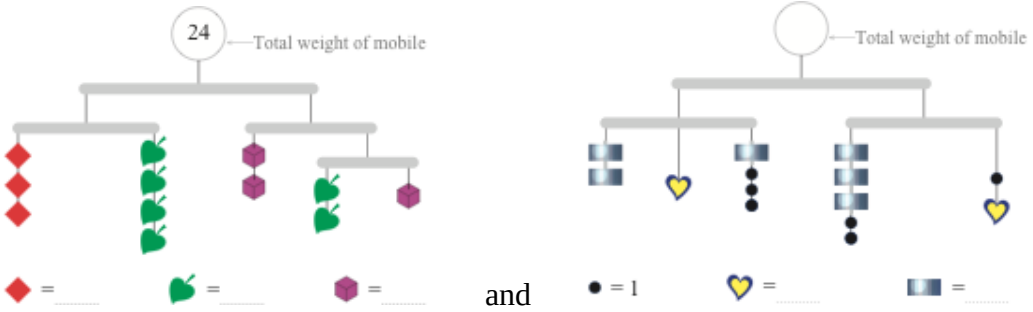
6 Note that this is not an epistemological statement: not a theory of learning, and not research result. This is a personal view of what mathematics is. If all one wants is for students to know the results of prior mathematical thinking, students may construct their knowledge from a much more predigested diet.

judiciously) past some ideas that they *could* invent on their own but that don't seem worthy of their valuable time or effort.

What about constructionism?

Another message from constructivism is about the richness of the soup out of which students construct their ideas. Lectures really *can* be excellent, even with children, if they are good enough. After all, a great movie is not "active" or "manipulative"—it is essentially a lecture with really good visuals—and it can be very effective, moving, and educational. And great story-tellers often don't even have such rich visuals. But "great" is hard to achieve, and lectures are a "thin soup." Listening, alone, may not yield as much information as listening *and* seeing. Moreover, because it *is* thin and requires intense concentration, it's cognitively taxing and so a listening-only lecture typically can't be long at all. When one manipulates some object and talks about the experiment, one not only sees and hears, but also feels and moves and creates words to explain. Perhaps more importantly, one controls what one is seeing, and is able to re-run the "movie" and "narrate it" in various ways, and hear others' interpretations, and get feedback on one's own. Building is typically social: we show and discuss what we build, and even where we do not volunteer that conversation, people *see* what we build and trap us into talking about it. The soup is just richer. There is more "stuff" in it from which to construct ideas and there is more redundancy of information. One is not so dependent on catching every detail in one way: the relevant information is available in *several* forms. More brain (whatever that means) is involved.

The thick-soup theory helps interpret the "ism" in constructionism. Constructionism could be a religion or political stand or even just a basic value, but then there's less to discuss: basic values cannot be challenged on logical grounds because they are *premises* for reasoning rather than conclusions of it. But a *theory* can be tested: does *constructing* things lead to "better learning"—darn! "better learning" needs a definition, doesn't it?—than just *consuming* things? In my work—development (construction) of curriculum—I start with the premise that constructing *is* more effective than mere consuming, but I'm quite eager to test that premise. We see that solving "mobile puzzles" like these



gives students a set of experiences and a platform of intuition from which they can build the logic for various algebraic "moves" that teachers often enough just deliver with little or no rationale. We don't yet have hard data that having the chance to *build* such puzzles gives students an even richer soup in which to analyze that logic, but it *looks* convincing. In a vastly more prosaic domain, solving a simple word problem like "Hiroshi has 3 marbles and Imani has 7 marbles. How many marbles do they have altogether?" may be of some value to students—note my tentative language!—but presenting the same problem *without the question* and asking students to construct good questions to ask gets much deeper analysis of the situation. Along with the question that the curriculum writer happened to think of, children ask things like "Who has more?" or "How many more does Imani have?" or "How many more does Hiroshi need if he

wants as many as Imani?" or (occasionally) "Could they share that total number of marbles equally?" Having the chance to produce the same kind of language they are expected to become competent consumers of helps not only the language learning, but even the more basic recognition that one situation can *have* more than one associated question!

I give these examples to illustrate that programming a computer or building a physical structure are not the only ways to *construct* rather than just consume in some educational environment, but it is not a surprise that programming makes a special contribution. Programming involves constructing on many levels—the object/behavior one is intending to create, an analysis of that object or behavior, and the "logical argument" (the algorithm) that creates it—and making the details of that construction explicit and precise.

And what about experimentation and exploration—and, for that matter, programming a computer—in class? These are inevitably less "organized" and "straightforward" and "clear" than a lecture: they are messy[7] and make it hard to have everybody in the same place. But I'd argue that we are not really in any less control over what gets into the students' heads, anyway. In fact, we get more opportunity to guess at what might be going on in students' heads when we can watch and listen to the students as they work than when they are quiet and listening to us. If we take the opportunity and pay real attention, students' active involvement puts us (often) in a better position to interact with them. Of course, stuff goes in when we lecture, too, but it can be harder to know what that stuff is until the test, and it's hard enough even then.

The moral of this story

The story of the thinking of one clever seven-year-old illustrates that, while teaching is certainly a position of power—power that we should be careful not to abuse—it is not a position of intellectual Omniscience or Omnipotence. Would that it were that easy. As for constructivism, it opens our eyes (once again) to what has been said long before the term came into vogue: minds are not buckets. Construc*tion*ism reminds us that, while self-construction of knowledge is the only game in town, the *public* building of viewable artifacts that are sharable with others supports the *mental* building of ideas in the privacy of one's own head by being a richer "soup" for that internal learning. As for teaching, constructivism suggests not so much a *replacement* of practice as a *broadening* of practice: lecture and hands-on both make sense, and experience and good judgment (along with some *theoretical* way to think about the potential advantages and drawbacks of each) can help one decide how to use both effectively.

Do I like holding this theory of learning? That's a bit like asking if I like being human. It's not as convenient as being a god, I suppose, but it is less terrifying, and it is more consistent with reality. And besides, in neither case am I offered any choice.

[7] This is hard for a teacher, but may not be so hard for children. Children live in a messy world and have less control of the environment around them than do adults. It makes sense, then, that they are adapted to make sense of and learn in that messy world, and to find pattern and order in whatever fragmentary and disorganized data they get. Children are constantly solving puzzles in their attempt to makes sense of the real world. This involves seeking structure, while ignoring some details. It is an act of abstraction that children naturally start with. (See, for example, Goldenberg, Cuoco, and Mark (2010), The Scientist in the Crib by Gopnik, Meltzoff, and Kuhl, and the ideas described by Stephen Pinker in How the Mind Works, a body of ideas deriving principally from research in cognitive science and interpreted in the light of evolutionary psychology.)

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