



## 3D in Excel

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### Abstract

*A method is presented of explaining the principles of 3D graphics through making revolvable and sizable projections of cube in Excel with solved problem of overlapping of the faces.*

### Keywords

*Excel, 3D graphics, projection, overlapping*

### Introduction

Virtual reality applies to environments that simulate physical presence in the real or imaginary worlds. 3D computer graphics constitutes the basics. The technology is used in computer games, films, education software, etc. Traditionally, virtual reality environments are developed by using programming. However, the equations that govern the transformation from 3D to 2D are easy to evaluate in spreadsheets. Creating a projection of a 3D figure in Excel is an interesting introduction to 3D graphics without any need for programming. In the workshop, a method is presented of explaining the principles of 3D graphics through making revolvable and sizable projection of cube in Excel with solved problem of overlapping of the faces. Orthographic parallel projection is used. In a constructivist way, students comprehend the principle of depicting 3D objects on the computer screen when creating the applications. A 90 min lesson comprising Sections 2 and 3 of this paper was taught nine times to 147 students of teaching informatics and applied informatics. All participants found the lesson interesting. Out of the 45 who had not understood the principle of 3D graphics before the lesson, 44 comprehended it in the lesson.

### Orthographic parallel projection

Let  $Oxyz$  be orthonormal right-handed coordinate system (Fig. 1). Let plane  $\rho$  be in distance 1 from point  $O$ . Let line  $p$  be going through point  $O$  perpendicularly to plane  $\rho$ . Let point  $O'$  be the intersection of line  $p$  and plane  $\rho$ . Then,  $O'$  is the orthographic parallel projection of point  $O$  onto plane  $\rho$ . Plane  $\rho$  is the projection plane, vector  $\mathbf{OO}'$  is the projection vector; its length is 1. Angles  $\phi$  and  $\theta$  give the position of plane  $\rho$  through vector  $\mathbf{OO}'$ . They go from  $-180^\circ$  to  $180^\circ$  and from  $-90^\circ$  to  $90^\circ$ . If  $\theta = 0$  and  $\phi = 0$ , then vector  $\mathbf{OO}'$  merges with axis  $x$  and plane  $\rho$  is parallel to coordinate plane  $yz$ . Then, let  $x'$ ,  $y'$  be the intersection lines of plane  $\rho$  and planes  $xy$ ,  $xz$ . If  $x'$ ,  $y'$  are scaled by the unit of axes  $x$ ,  $y$ , starting at point  $O'$ , then coordinate system  $O'x'y'$  is defined in plane  $\rho$ . Let  $A(x, y, z)$  be point in space. Let line  $r$  be perpendicular to  $\rho$ , that is, parallel to  $\mathbf{OO}'$ , and going through point  $A$ . Intersection  $A'(x', y')$  of line  $r$  and plane  $\rho$  is the orthographic parallel projection of point  $A$  onto  $\rho$ . It holds that (Benacka, 2008)

$$x' = -x \sin \phi + y \cos \phi, \quad (1)$$

$$y' = -x \sin \theta \cos \phi - y \sin \theta \sin \phi + z \cos \theta. \quad (2)$$



If we identify plane  $\rho$  with the computer screen, then we can project any 3D figure on it provided we know the  $x, y, z$  coordinates of the vertices.

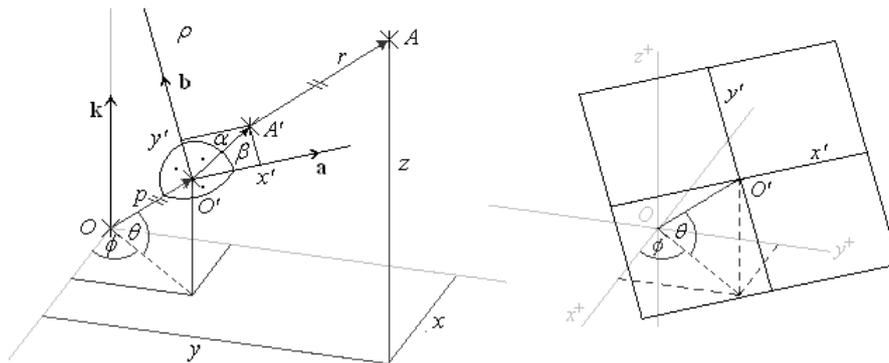


Figure 1. Orthographic parallel projection  $A'(x', y')$  of point  $A(x, y, z)$

Remark: When the author explains this theory, he uses a transparent plastic sheet as the projecting plane, a wire model of the  $xyz$  coordinate system, and two wires as vector  $\mathbf{OO}'$  and the projection direction. The meaning of angles  $\phi$  and  $\theta$  is made clear by moving the system of (sheet + wire for  $\mathbf{OO}'$ ) in the model of the  $xyz$  coordinate system. Then, a carton cube is placed in the  $xyz$  coordinate system so that the wires for axes  $x, y, z$  are inserted through the holes in the centres of the faces (Fig. 2a). The system (sheet + wire for  $\mathbf{OO}'$ ) is set up so that angles  $\phi$  and  $\theta$  are about 30 and 20. Then, the vertices of the cube are projected on the sheet using the wire for the direction. Linking the projections of the vertices properly gives the projection of the cube.

## Projecting a cube in Excel

Let ABCDEFGH be a cube of edge length  $a$  with the centre at origin  $O$  of  $xyz$  coordinate system and faces parallel to the coordinate planes (Fig. 2a). The application that projects the cube in Excel without solving the overlapping of the faces is in Fig. 2b. Edge length  $a$  is in cell C6. Angles  $\phi$  and  $\theta$  are in cells C4 and C5 in degrees, converted to radians in hidden cells J4 and J5.

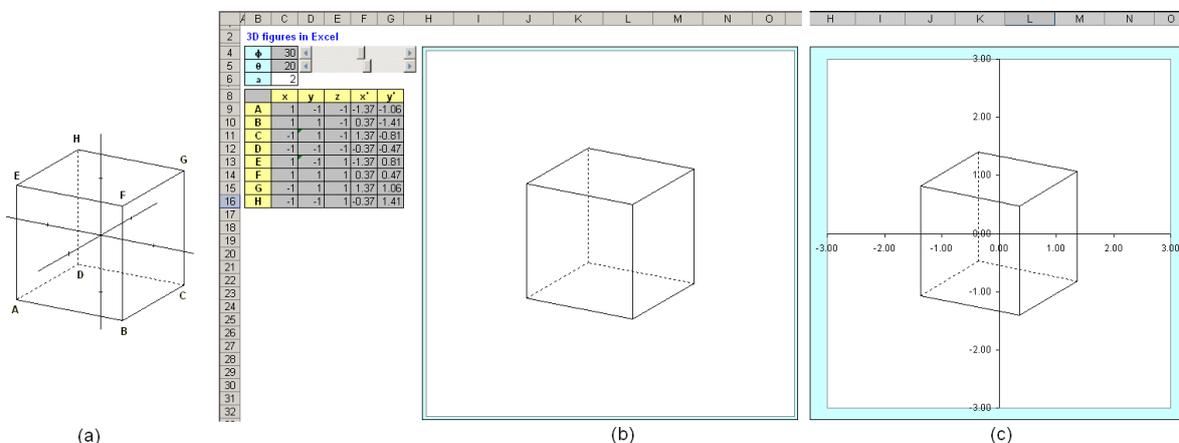


Figure 3. Cube in  $xyz$  coordinate system (a); cube with non-solved overlapping of the faces, the chart axes switched off (b) and on (c)

The  $x, y, z$  coordinates of the vertices are calculated in C9:E16 referring to C6, which makes the projection sizable. It is  $C6/2, -C6/2, -C6/2$  for point A. The  $x'$  and  $y'$  coordinates of the vertices



are calculated in F9:G16. Each edge is drawn as a two-point xy line graph. There are 12 graphs for 12 edges. If an edge becomes invisible when the real cube is rotated, the user can double-click the edge in the chart and change it to dash or white to make it invisible. The maximum and minimum of axes  $x'$ ,  $y'$  are set to  $-3$  and  $3$ . Then, the axes are switched off (Fig. 2b, 2c). Angles  $\phi$  and  $\theta$  are governed by scrollbars. For the first one, LinkedCell is K4, Min is 0, Max is 360, SmallChange is 1, and LargeChange is 5. Then, cell C4 is rewritten by the formula  $=K4-180$ . That makes  $\phi$  go from  $-180^\circ$  to  $180^\circ$  by  $1^\circ$  or  $5^\circ$ . For the other scrollbar, LinkedCell is K5, Min is 0, Max is 180, SmallChange is 1, LargeChange is 5. C5 contains the formula  $=K5-90$ .

### Solving the problem of overlapping of the faces

The application that projects the cube with solved overlapping of the faces is in Fig. 3. Each face is drawn as a 5-point xy graph going through the vertices of the face where the fifth point merges with the first one. It holds that if a face becomes invisible when the cube is rotated, then the angle between its normal vector  $\mathbf{n}$  oriented out of the figure and the projection vector  $\mathbf{s} = \mathbf{OO}'$  becomes bigger than  $90^\circ$ , so the scalar product  $\mathbf{SP}$  of the vectors becomes negative. On that condition, the face can be made invisible by letting it collapse into point 0 by using function IF. The faces are calculated in separate ranges. The front face is in range B20:E26. Vector  $\mathbf{s}$  is calculated in C18:E18. The vertices are in B22:B26. Normal vector  $\mathbf{n}$  is in C20:E20. Its scalar product  $\mathbf{SP}$  with vector  $\mathbf{s}$  is in cell E22. Cell C22 and D22 contain the same formula  $=IF(AND(\$P\$13,\$E\$22<0),0,VLOOKUP(B22,\$B\$9:\$G\$16,5))$  except for ending  $\dots,6))$  in D22. The formulas are copied down. Cell P13 contains the value true or false of CheckBox "Overlap". The overlapping works if the CheckBox is checked. The back face is in B28:E34. It is invisible in the left side of Fig. 3 (note the checked "Overlap", the negative  $\mathbf{SP}$ , and the null  $x'$ ,  $y'$ ) but visible in the right side (note the unchecked "Overlap", the positive  $\mathbf{SP}$ , and non-null  $x'$ ,  $y'$ ).

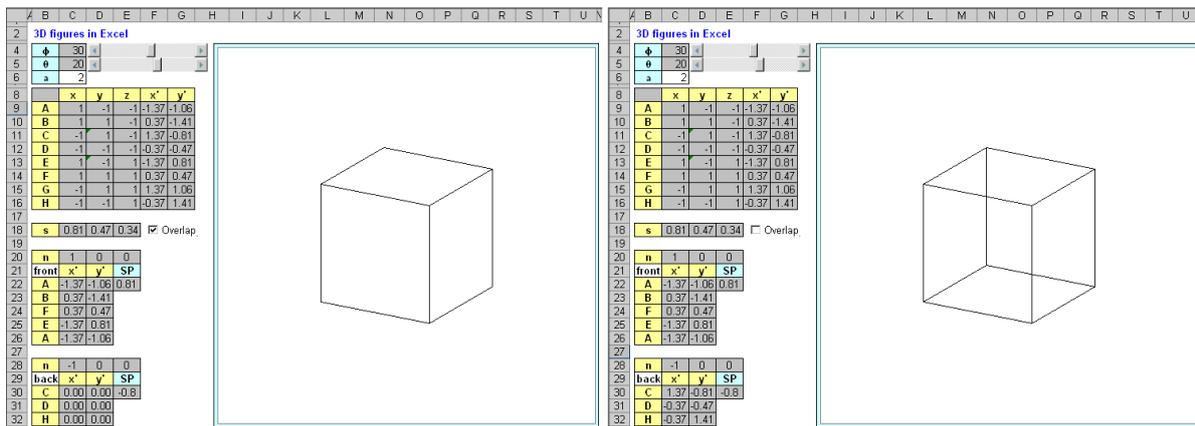


Figure 3. Cube with solved overlapping of the faces

### Acknowledgement

The author is a member of the research team of project PRIMAS (Promoting Inquiry in Mathematics and Science Education across Europe) funded by the EU 7<sup>th</sup> Framework Programme, grant agreement 244380.

### References

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