Meanings for 3d mathematics shaped by on-line group discussion

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Abstract

Several attempts have been made to design and study social aspects of constructionist learning that built on the idea of distributed cognition or collaborative knowledge-building within an on-line community of learners. Here we describe designs and activities of a European project titled Metafora (from meta-forum), where the focus moves to learners' reflections on their group work, a kind of meta-cognitive group think. The paradigm was to encourage students working communally to learn how to learn with and from each other as they work to collaboratively address challenging problems in science and mathematics. For this purpose, a web-based digital system that supports this "learning to learn together" (L2L2) approach is being developed. We study students' meaning making processes in this context and the ways they interacted with a beta version of the system, as they worked with an incorporated half-baked microworld in 3D Turtleworlds consisting of a buggy 'Twisted Rectangle'. We try to show how students' meanings were shaped and influenced by learning to learn together aspects that came forward as they shared and discussed their ideas and plans through the system L2L2 support tools.

Keywords

3d geometry, collaborative meaning generation, on-line discussions, learning to learn together

Collaborative constructionist activities

The idea of collaborative constructionist activities that entail discussing and sharing artefacts using computer networks or more contemporary web tools, has long been raised in the constructionist community. Noss and Hoyles studied meaning making in situations where students worked in pairs talking to each other while working on a problem with one computer (1996). Resnick (1996) introduced "distributed constructionism" based on (Salmon's, 1991) "distributed cognition" to study online communities working with constructionist media. In this new paradigm, communal knowledge building constitutes the core of the community's activities and takes place as the students discuss about their constructions via emails or fora, share them (or parts of them) through web versions of microworlds and even collaborate real-time in the process of designing and creating new artefacts.

Taking the idea to the World Wide Web in the last years, several projects have supported these kinds of activities. The CoLabs Project (http://matchsz.inf.elte.hu/Colabs/) was launched with an aim to develop tools for supporting collaborative model building through an e-learning platform both in an asynchronous and a synchronous mode. Imagine Logo microworlds (or parts of them) created by the students could be uploaded in the CoLabs Platform to serve as resources for brainstorming or further microworld development for others and as objects of discussion in

community forums. On-line game playing with browser-run Imagine Logo microworlds, where connected students could work in a shared common space as well as in private ones, was also an option in the CoLabs platform (Kalas & Winczer, 2006), allowing the creation of artefacts mutually shaped by the community of students.

The Weblabs Project (http://www.lkl.ac.uk/kscope/weblabs/index.htm) also focused on building a web-based system (WebReports) for collaborating, co-constructing and thinking about models the students shared within the community of users (Noss, 2004). The models were created in ToonTalk (Kahn, 2004), a programmable environment in which the students, acting as avatars in a virtual world, operate animated cartoon characters of the microworlds like robots, birds and helicopters. In the WebReports system, the students, apart from sharing their ideas, describing in a textual form and discussing their experiences with their individual or group work in the microworld, could also share animated ToonTalk models, bringing into the core of the activity the question "how does this model" works. The co-construction of models came as the students reported on the their mathematical understandings in the process of training their robots to carry out specific actions, discussed and argued about their validity, reflected on their emerging mathematical ideas and redesigned their constructions.

In these cases students' constructionist activities embedded the "social" aspects of learning that emerged from community work. The students collaborated to shape a common artefact by discussing about it, sharing and customizing it. Working in a group, however, poses new challenges and potentials for learning. Issues of "how are we getting on as a group", "what are the roles", "what are we achieving as a whole" influence understandings and progress. Let us call these issues "socio-metacognitive" in that they refer to how students not only engage in collaboratively generating meanings about mathematics and science concepts, but also engage in learning how to learn together, in learning how to learn with and from each other as they work in groups to address challenging problematic situations in science and mathematics.

The Metafora Project views computer-supported learning in groups as a complex task that requires from students as they collaborate to also become aware of elements considered to be important for successful learning in collectives and to learn how to put those elements in use (Wegerif & Yang, 2011). When working in a collective task, the group members need to be able to show distributed leadership, planning and coordinating the tasks each member needs to carry out, motivate one another, ensure engagement for everyone (or react when this doesn't happen), reflect on the quality of the work delivered at an individual and collective level through peer reviewing, deal with constructive criticism, reflect on the overall direction of their work (devising help-asking and help-seeking strategies when needed), and make sure all group members are doing what is expected from them (Wegerif et al., 2012). All those elements constitute the key components of the "learning to learn together – L2L2" pedagogical approach adopted by the project. A web-based system that includes a planning, a discussion tool and a set of microworlds- was being developed at the time of writing with the intend to raise students' awareness in the process of learning together when working in groups and to facilitate them in putting to use their understandings of the key components of learning together.

In this paper we discuss a mathematical group activity with a half-baked microworld in a 3D version of Turtleworlds, our own Turtle Geometry medium integrating dynamic manipulation-execution of variable procedure values (Kynigos, 2007). We called the microworld the "Twisted Rectangle". Students were given a buggy procedure to construct a rectangle where one of its segments twists along a plane vertical to the one it belongs to when it's not twisted.

Such buggy procedures are given to students as half-baked microworlds in the sense that they

potentially construct something interesting but are incomplete or faulty by design. The point is to invite students to deconstruct them, build on their parts, customize and change them in a personally meaningful to them way, engaging in the way in meaning-making processes. These microworlds have been perceived as 'boundary objects' (Kynigos, 2007) i.e. questionable and improvable objects engaging members of different communities in meaning making emerging from the joint de-bugging effort. They may operate as a tool of communication, around which, the members of the community structure their activities. Thus, in this case, the meanings generation processes are considered to emerge and be shaped both by the students' mathematical activity as they interact with the half-baked microworld and their social activity as they discuss on how to change and customize it.

So, we studied middle school students' interactions with the "Twisted Rectangle" microworld (TwR), focusing on their on-line discussions, as they tinker and try to fix it and as they subsequently (after fixing it) create funky constructions using the correct TwR as a base. We put emphasis on how the students' actions within the microworld were specifically shaped by their need to articulate their own ideas to others and by the ideas brought at the table by other group members. Moving between on-line group discussions and microworld actions, we seek to identify L2L2 skills such as organizing and coordinating the work so as to proceed as a group, discussing and evaluate findings from others, reflect on own findings, devise help seeking and help offering strategies.

The Digital Tools

The 3d Math Authoring Tool

The "3d Math" Authoring Tool (http://etl.ppp.uoa.gr/malt) is the latest web-based version of MaLT, built with Unity 3D a multiuser web gaming engine. MaLT is a 3D version of Turtleworlds, our own Logo-based Turtle Geometry medium. It integrates dynamic manipulation of variable procedure values so that the effect is like a Dynamic Geometry System as a value or combination of values change (Kynigos & Psycharis, 2003). MaLT allows the creation, exploration and dynamic manipulation of 3d geometrical objects, graphically represented inside a 3d virtual space.
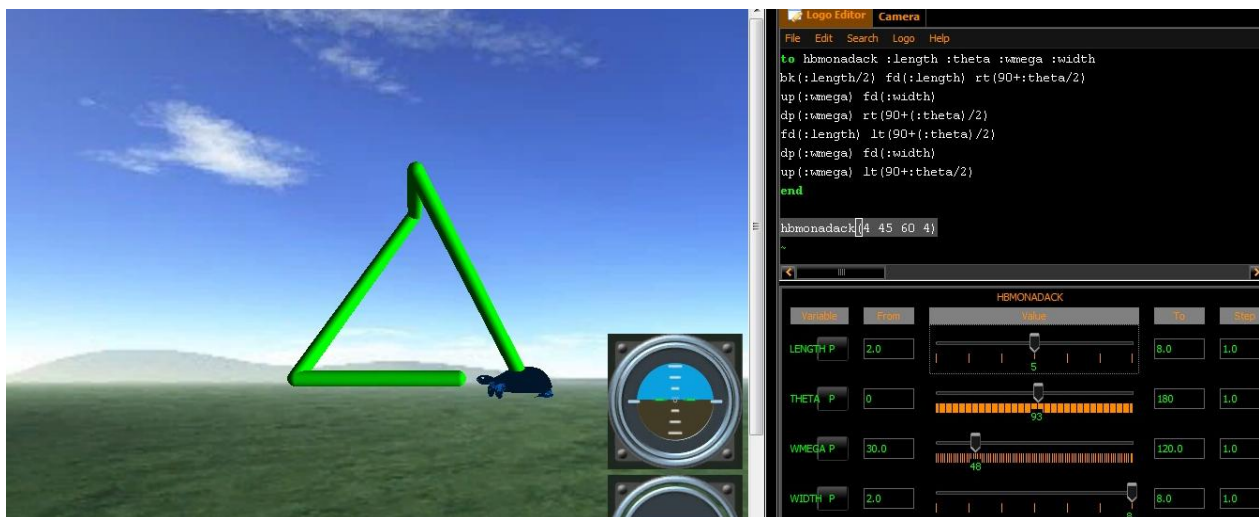

Figure 1: The Twisted Rectangle half-baked microworld

Building and manipulating geometrical objects in the 3d Mathematics Authoring Tool, however,

is not restricted to solely looking at the 3d world form static 2d orthographic views. A 3d Camera Controller gives students the opportunity to navigate around, inside and through their constructions, offering the potential for new ways of visualizing 3d space and conceptualizing mathematical notions, especially ones related to stereometry (Latsi & Kynigos, 2012; Moustaki & Kynigos, 2010). We view 3d Math as an authoring tool for developing half-baked microworlds (Kynigos, 2007), such as the "Twisted Rectangle" (Figure 1).

The Metafora System

Being a completely web-based environment, 3d math is fully embedded in the Metafora System (Dragon et al., submitted). Metafora System is an on-line software platform that offers a set of microworlds for exploring genuinely challenging problematic situations in science and mathematics and shared workspaces, within which the students may discuss their ideas, argue, negotiate and engage in joint decision-making processes.

One of those shared workspaces is the Planning Tool. Within the Planning Tool, the students working together, form a common Plan depicting the course of action they should take so as to explore and address the problematic situation at hand. Assigning roles to the group members and allocating tasks is also part of the Plan to be created, as the students need to think as well about the way they should collaborate to reach their mutual goal. To form their Plan, the members of the group select and place on the shared workspace cards that correspond to Activity Stages, Activity Processes, Roles and Attitudes.

The other shared workspace is LASAD, a Discussion Tool that allows communication among individuals or groups of students. The members of the group place in LASAD's UI text boxes with their ideas (we call those "contributions") and link them with existing ones, forming in this way a kind of discussion map (Figure 3). Different types of pre-defined contributions are available and the students choose which one to insert from the available library. To further tag each contribution with respect to its content (e.g. tag a contribution as "a suggestion" or as "a claim"), a dropdown list is available in each text box. Thus, the LASAD Discussion Tool may also serve as reflection space for the students, as when they make public an idea, they also need to be explicit about what is that this idea brings to the discussion that takes place.

Research design and methodology

Our research approach was based on the idea of studying learning in authentic settings through "design experiments" (Cobb et al. 2003). "Design experiments" aim to contribute to the development of grounded theories on "how learning works" and are conducted with the intention to shed light on the relationships between the material designed for the experiment (usually innovative technological artefacts having added pedagogical value) and the learning processes within a specific context of implementation.

Context and participants

The Study described in this paper took place in a Lower Secondary Education School in Athens (1st Experimental Middle School of Athens) with ten 9th grade students (14 years old). The students worked together at the school's lab for 13 Sessions (26 school hours) in two Groups. Group A consisted of three Subgroups and Group B of two Subgroups of two students each. Each duet of students shared the same PC and used the LASAD Discussion Tool when working on-line with the microworld and with the Planning Tool so as to communicate thoughts and ideas across the different Subgroups. The Groups communicated with each other in face to face meetings.

The Educational Technology Lab (ETL) researchers, adopting a "participant observation" methodology, chose not to intervene in the experimentation so as to give out specific instructions or provide the "correct answer" to the students on how to address the challenge and proceed. They preferred to pose meaningful -often intriguing- questions at certain time points, so as to encourage students to continue their explorations, elaborate more on their thoughts, share and discuss their ideas collaborating with the other students.

Tools and Tasks

Phase 1: The Twisted Rectangle microworld

For this Study, we designed a half-baked microworld (Kynigos, 2007), called the "Twisted Rectangle". The Twisted Rectangle is a skewed quadrilateral as one of its segments twists along a plane vertical to the one it belongs to when it's not twisted. Running, however, the Logo program in this microworld, the figure depicted on screen is not a closed, but an open one, as the end of one of the rectangle's sides, is not attached to the rest of the shape (Figure 1). The students working in two Subgroups of two students each, were asked to try to "make the shape close". Being a half-baked microworld (Kynigos, 2007), the "Twisted Rectangle" that invites students to deconstruct its parts so as to make sense of what causes the "buggy behaviour". As we didn't intend to provide any answers on how to achieve this goal, but ask students to discuss their ideas in an inter and intra Subgroup mode, we had already prepared a discussion space in LASAD in which the Subgroups could meet and share their findings as they explored this issue within the Twisted Rectangle microworld.

Phase 2: Constructions with the Twisted Rectangle as a building block

At this phase of the Study, we asked the students to create their own constructions using the closed Twisted Rectangle as a building block. The students working in Subgroups were expected to discuss their ideas and share Logo code or parts of their constructions so as to create meaningful for them complicated artefacts.

Data collection-Method of analysis

A screen-capture software (HyperCam2) was used to record students' interactions the Metafora Tools, together with their verbal interactions. Since previous work with 3d Math had shown an extensive use of gestures as means to explain and communicate turtle movements and turns, a Camera was added to record students' hand and body movements. The corpus data is completed by the students' LASAD and Planning Tool maps and the Researchers' Fields notes. The video-recorded data from the screen-capture software were verbatim transcribed, while the rest of the data were used for providing additional details. In analysing the data, we searched for verbal exchanges between the students and interactions with 3d Math and LASAD that indicated that learning to learn together aspects were brought forth as they students attempted to address the challenge when working with the half-baked microworld.

Results

The episodes of this section are selected so as to highlight the students' interactions and describe: 1) the intra-Subgroup discussions as the students explored their ideas in the "Twisted Rectangle" microworld and 2) the inter-Subgroup discussions in LASAD around their findings. We draw our attention, however, on how the students' actions within the microworld are specifically fuelled: a) by the fact that they need to articulate their own ideas in LASAD and explain them to the other Subgroup as explicitly as possible and b) by the fact that they are receiving an idea from the other

Subgroup which they need to try out and decide on its feasibility and usefulness in the process of closing the figure in the "Twisted Rectangle" microworld. The episodes described in this section start with students viewing the Plan they have created at the Planning Tool.

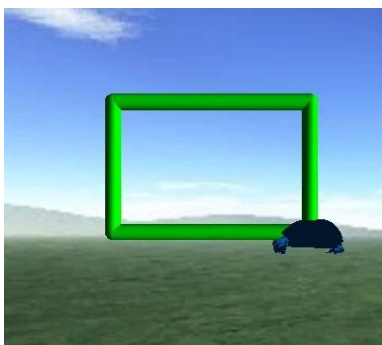## Enacting the "Discuss Findings" Activity Stage – Planning Tool (Figure 2)

The students of Group A have created a common Plan on how to address the challenge of closing the Twisted Rectangle. Having "Formed the Hypotheses" to test and having spent quite sometime "Experimenting" working with the microworld separated in two Subgroups ("exploring questions" and "gathering information according to their observations"), the students of Group A reach the point at which they need to "Discuss their findings" (Figure 2). To enact this Activity Stage of their Plan, the two Subgroups of Group A move to LASAD to share the ideas, "observations" and "conclusions" coming from their explorations with the microworld.

Figure 2: Part of the Plan the students have created to address the challenge

Trying to close the Twisted Rectangle - 3d Math

The students of Subgroup No1 have made several attempts to close the figure, mainly by manipulating the sliders and replacing the initial Logo program's variables with specific values. The idea they come up with is that "no matter if we change the length of the Twisted Rectangle's sides, the shape will always close if "wmega" is equal to 90 and "theta" is equal to zero". Being sure about this idea, they decide to share it with Subgroup No2 and start typing it in LASAD (Contribution no2 – Figure 3). Writing it down, however, and talking about the exact wording to use, one of the students seems to express second thoughts on the validity of this idea. As the shape that appears on the screen looks like a 2d rectangle, she points out that they haven't really addressed the problem in all cases, but have managed to just close the shape in a particular situation.

Comparing the shape that appears on the screen with the Twisted Rectangle (Figure 1), the students come to the conclusion that what makes the shape being a 2d one, is the fact that two of

triangles that appeared on the initial shape were either eliminated or turned to right. This also changed tangent, cosine and sine values that should be relevant to creating the correct triangles so as to close the shape. This final idea, although not implemented or fully explored, is also added in LASAD by Subgroup No1 (Contribution no2 – Figure 3).

Sharing strategies - Communicating through LASAD (Figure 3)

The students of Subgroup No2, after asking Subgroup No1 for further information on how to implement their idea (Contribution no3), they go back to 3d math to test it for its feasibility and validity. What bothers them is that the shape generated with theta equal to 0 is a 2d one (Contribution no10), and express strong objections on if and how Subgroup No1's idea could help the Group in achieving the common goal of closing the Twisted Rectangle. The do try it out, however, in 3d math also taking into consideration the explanations the students of Subgroup No1 give as they pursue and try to refine their idea within their microworld. The students of Subgroup No1 work with the 2d rectangle (which they believe is a special case of the Twisted) and try to identify triangles (right ones as the shape is a rectangle), that could help them in defining relationships between the shape's angles and the side lengths (Contribution no14).

The outcome of this process is for the students of Subgroup No2 to move one step further. They claim that what makes the figure "non-square" or "non-uniform" in general, is the value attributed each time to the theta angle. Furthermore, they suggest that "theta" is not a visible angle (e.g. an angle between two of the Rectangle's sides), but one that is between one of the sides and some auxiliary line they need to draw (Contribution no15). As this angle is not visible, the students of Subgroup No2, decide that the best way to show Subgroup No1 this finding is to meet face to face.
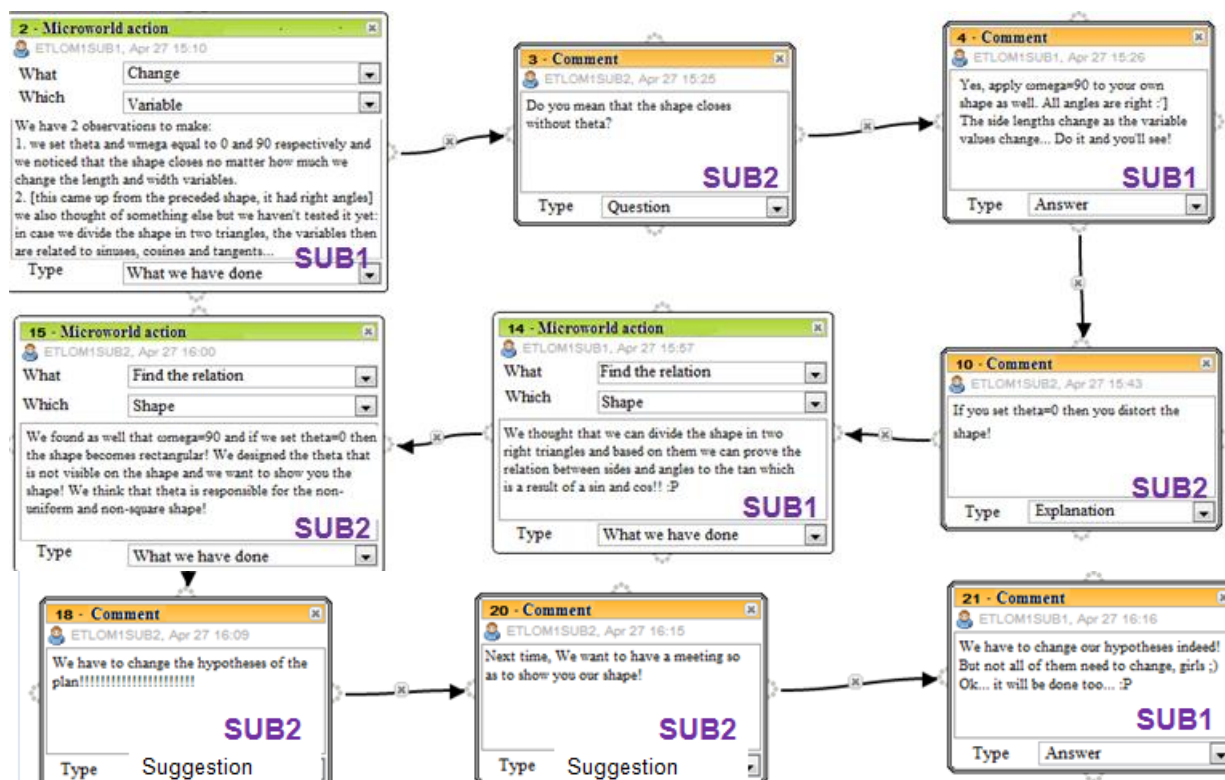

Figure 3: Subgroup No1 and Subgroup No2 discuss around their strategies on how to close the TwR

The students of Subgroup No1 receiving Contribution no 15 from Subgroup No2, move to 3d

math and try to verify if "theta" causes this "non-uniformity". They giving a constant value to the "wmega" variable and manipulate the "theta" slider to attribute it different values.

As the hypotheses they had come up with originally in the Plan they had created (Figure 2), seem to be quite obsolete now, both Subgroups agree that they need to revisit their hypotheses and update them according to their new findings with in 3d math (Contributions no18, 20 and 21).

Creating their own constructions using Twisted Rectangle as a building block

The students of both Subgroups after managing to close the figure (the students of Subgroup No2 do it first and explain Subgroup No1 the strategy they followed), they both attempt to use the closed Twisted Rectangle as a building block to create their own artefacts.

Subgroup No1 works with creating a "flower" made up of Twisted Rectangles. Each Twisted Rectangle is created by placing the Turtle in a specific location in 3d space and turning it an amount of degrees with respect to the previous Twisted Rectangle. The number of times to repeat the Twisted Rectangle so as to create the flower's petals is defined by a variable.
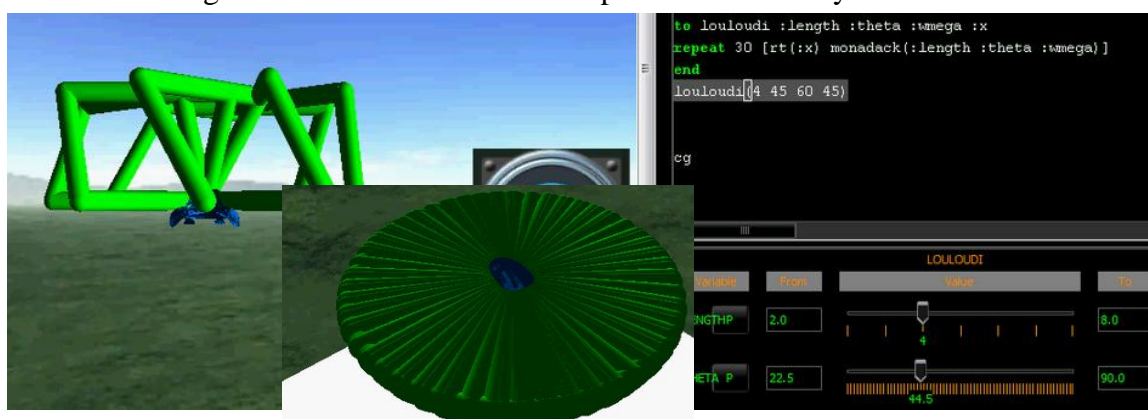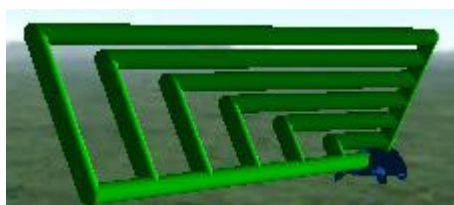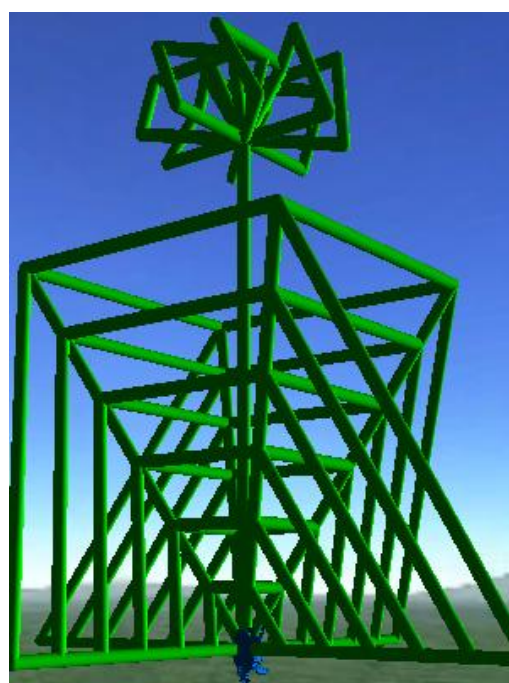

Figure 4: Subgroup No1's "Flower" using the Twisted Rectangle as a building block

The students of this Subgroup notify the students of Subgroup No2 about their construction and share through LASAD the Logo code of the "flower" procedure. The students of Subgroup No2 have been working on a construction that includes several Twisted Rectangles all starting from the same point in 3d space but having different sizes (*Figure on the left*). That gives the impression the one Twisted Rectangle is situated inside the other.

Receiving the Logo code from Subgroup No1, the students of Subgroup No2 create a new Logo procedure that includes their own construction, which is turned to a "vase", and the "flower" procedure. This new construction is called "flower with a stem" (*Figure on the right*). The original "flower" procedure is used, however, by Subgroup No2 with constant values, instead of variables. This causes Subgroup No1's reaction, as soon as Subgroup No2 makes the new "flower with a

stem" procedure public through LASAD. The students of Subgroup No1 create another procedure, called "the great flower", in which they use two of the procedures that Subgroup No2 has created and included in the "flower with a stem" as well as the "flower" procedure with variables instead of constant values.

Discussion

The students of Subgroup No1 and Subgroup No2 are both given the Twisted Rectangle half-baked microworld and are asked to make the open 3d figure a closed one. The two Subgroups work independently with the microworld but share mutual workspaces in the METAFORA Platform. They use the Planning Tool to create a Plan to define the course of action to be taken so as to achieve this common goal as one Group and LASAD to discuss their ideas around their 3d math explorations. Taking a close look at the students' moves between their microworld, their LASAD discussion map and their Plan, we seek to identify L2L2 elements that may influence the students' meaning generation processes within the microworld. Our focus is not only in their actions with the tools (microworld, LASAD, Planning Tool) per se but also on their within Subgroup discussions as they decide which actions to carry out in these tools.

Our findings indicate that the meaning generation processes within the microworld are fuelled by the dialogue between the two Subgroups in the LASAD tool. This dialogue is sustained by the fact that the students constantly move between their discussion map and their microworld, trying out ideas and making them objects of discussion and reflection both for themselves and for the students of the other Subgroup. Each time a new idea towards achieving the common goal (closing the figure) is proposed by a Subgroup, the members of the other one try it out so as to evaluate and check it for its usability with respect to the Group's goal. At the same time, the members of the Subgroup initially suggesting the idea, revisit it and come to reflect on it so as to make it more explicit for the others (which demand explanations if they feel that they don't understand the details). In the case of Subgroup No1, this resulted in extending the original idea, offering new insights on how to implement it. Reflecting on both approaches (their own and the one offered by the students of the other Subgroup), the two Subgroups come to put their ideas not only under peer assessment processes but also under self-assessment processes.

Subgroup No2, being proactive, use the feedback and experience from their own and the other Subgroup's explorations and taking control of their learning as a Group, decide that they need to meet face to face so as to explain to the others where "theta" angle is. As they had drawn an auxiliary line on paper, they feel they can't easily explain to the others how to do it on-line and decide to ask for a face to face meeting. In the same meeting, both Subgroups decide they need to revisit the original hypotheses they had made and report in their Plan the results of their try outs with 3d math. It seems that both Subgroups suggest that in order to proceed with their explorations as a Group (and thus engage in further meaning making processes), they need to share their understandings about the theta angle and redesign their course of action according to their up-to-that-point findings. Meaning generation, in this case, is fuelled by the fact that the students evaluate and monitor the progress they have made as a Group towards the common goal, assess specific learning outcomes as important for the Group's understandings and re-organise and re-plan the course of action to be taken so as to coordinate their further explorations with 3d Math.

The last part of the Results refers to students' constructions using the Twisted Rectangle as a building block. The students work in Subgroups without initially having a common goal which they need to achieve as one Group. Having worked, however, together to make sense of the

Twisted Rectangle's properties, they continue acting as one Group. Apart from ideas, in this phase of their experimentations, the students make object of reflection among the members of the Group also tangible constructions, which they share through LASAD in the form of Logo programs. The students exchange Logo codes and integrate the ones they receive in their own constructions. Meaning generation, in this case, is influenced by the fact that the students have integrated "sharing ideas and artefacts" within a Group as a "way of acting" for progressing the Group's work and learning.

Acknowledgements

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