Exploring the cone through a half-baked microworld

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Abstract

This paper represents the first part of a didactical proposal that investigates the construction of the geometrical solid of the cone formed each time through the motion of a different geometrical flat shape and thus leading to three different approaches to its properties. This results to having each time a different unit construction-generatrix. Grade 9 pupils after exploring the different types of geometrical solids the half-baked microworld depicts fix it, in order to permanently construct a cone. Pupils work in pairs using a specially designed computational environment that combines multiple representations of geometrical objects and their dynamic manipulation. The environment in which the pupils will work communally is that of constructionism.

Keywords

Cone, angle, slant angle, turn, generatrix, half-baked microworld, constructionism

Introduction

According to Kynigos (2007a) the term half-baked microworld describes a microworld that is explicitly designed to engage its users with changing it as the main aspect of their activity. An opportunity to explore, experiment, test and check conjectures, interpret and form mathematical ideas is offered through the *investigation* of half- baked microworlds. While students are trying to fix their given incomplete program resulting from the removal /modification of one of its items, they discover the mathematics in practice through the experiential activities that the half-baked microworlds constitute.

Difficulties that may arise for the pupils during the investigation of the cone:

1. Are related to the nature of computer programs in general as well as 3-D computer programs. Students during the investigation process tend to use mathematical concepts together with non-mathematical ones, without being fully aware of their existence and meaning (Kynigos, 2006).
2. Stem from the former mathematical knowledge of the students, the cognitive domains where they encounter difficulties in the traditional process of teaching, as well as the overall context of teaching and learning: in the students' cognitive schemas the concept of the angle referred to as a dynamic quantity, i.e. as turn, does not exist due to its static approach in the Greek curriculum.
3. With a combination of the above mentioned: especially the concept of the angle as slope is in itself a source of difficulty for the students, as the one arm of the angle is not apparent. As a result the students have further difficulty in pinpointing its correlation with the distance that the turtle has to cover so as to always form the geometrical solid of the cone.

On the other hand, research shows that when the angle is considered from its dynamic perspective, namely turn, as in MaLT, it helps to deal with common errors like "*students tend to measure the arms of the angle when comparing them*" as it allows pupils to see the angle as turn (Hoyles & Noss, 1987).
These are some of the findings that were considered in the design of the half-baked microworld.

Theoretical Framework

The necessity to bring to light methods and tools for a widespread and substantial teaching practice different from the traditional classroom teaching models dominated by the relationship teacher-transmitter quantity of information on the one hand and pupil receiver on the other, are of the most well-grounded in education issues. This dates back to the time of Piaget, about 5 decades ago.

Moreover, Papert (1980) argues when children talk to the computer in a language it comprehends, they may learn more of what they should be learning of mathematics than they can in classrooms without computers. Talking to the computer through programming enables them to think aloud of the way they think and somehow learn to face problems that they pose. The computer enables the child to link his or her experience thoroughly with fundamental mathematical ideas we want children to learn.

Constructionism is a theory which combines constructivism, that gives priority to the individual's active engagement, with the computational tools, that allow us to understand in depth the children's mental processes. Moreover, it is argued that students learn when they are actively engaged in situations that have meaning to them (learning by doing). Pupils learn only when their interest is aroused. And that happens when the situation is such that calls for investigation.

The constructionist theoretical perspective of the present study is based on the assumption that programmable geometrical constructions designed to help children abstract the notion of turtle movement in the 3D space provide a useful environment, due to the didactical engineering that it allows students to do with its representation tools and functionalities, for developing their conceptualizations of geometrical objects, like the cone.

The teaching proposal

The teaching proposal for the construction of the geometric solid cone from 9 grade pupils is developed through three different approaches of its properties, subsequently determining the corresponding structure unit- generatrix: 1. Iterative process of construction of line segments, with a specific slant angle, to the plane of the polygonal base around which they revolve- the construction unit is the segment 2. A set of circles lying on equidistant and parallel planes to the base cone –generatrix, of which the radius decreases in a constant rate until it becomes zero (0) and 3. The rotation of a right -angled triangle (generatrix) around one of its short sides.

Due to space considerations in this paper is presented only the first part, which is expected to last 4 didactical hours. The 14-year-old pupils work in groups of two or three in the computer lab, using a version of Logo developed by *MaLT (MachineLab Turtleworlds)*programming environment. So, the pupils are presented the notation code of the half-baked microworld (see Table 1, left column).

to half_baked :a :b :y repeat 36 [rt(10) fd(:y) gen_1(:a :b :y)] end half_baked (180 -0.3 0.4) to gen_1 :a :b :y bk(:y/2) rt(90) up(:a) fd((36/6.28)*(:y)/cos(:b)) bk((36/6.28)*(:y)/cos(b)) dp(:a) lt(90) fd(:y/2) end	to cone_1 :a :y repeat 36 [rt(10) fd(:y) gen_1(:a :y)] end to gen_1 :a :y bk(:y/2) rt(90) up(:a) fd((36/6.28)*(:y)/cos(:a)) bk((36/6.28)*(:y)/cos(:a)) dp(:a) lt(90) fd(:y/2) end	to cone_2 :a :x :y :n repeat :n [rt(:x) fd(:y) gen_1(:a :y :x)] end to gen_1 :a :y :x bk(:y/2) rt(90) up(:a) fd ((:y*360/:x)/(6.28*(cos(:a)))) bk ((:y*360/:x)/(6.28*(cos(:a)))) dp(:a) lt(90) fd(:y/2) end
Half-Baked notation code	Notation code after Debugging (a version)	Notation code after the extension

Table 1. Notation Codes

The graphical representation of the half- baked microworld that students watch in the 3-D Turtle Scene when they run the procedure is the image of a "sun" lying on the plane.

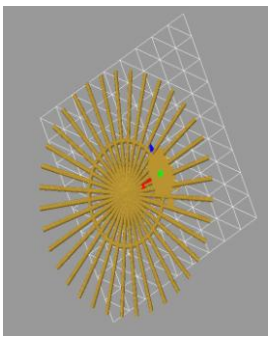

Figure 1. What geometric solids does the turtle construct?

The aim is threefold: 1. to investigate all possible solids that the turtle constructs 2.To tinker with the microworld to construct a cone (see Figure 1) and 3. To present an as much as possible refined the symbolic code, namely allowing the best possible approach of the cone.

In order to achieve the first goal while the basic constructive unit of the solids is hidden, the slant

angle, the students have in the beginning to detect it. As a result, in this initial stage, they will have to investigate angular relationships. In this point, the students are expected to find it difficult to distinguish them, since the visual representation of the half-baked code in the Logo Scene in no way resembles to a 3-D geometrical object. As students are engaged in navigating the turtle in MaLT they are expected to gain a sense of the mathematical ideas related to the construction of the different 3-D geometrical shapes (cylinder, truncated cone and bicone) i.e. pinpoint that (:a) is the parameter in the program that determines the shift of the turtle on the screen (slant angle), namely *angular relationships.*

Regarding the second goal: Students while experimenting with the sliders of the parameters (:a), (:b) and (:y) in *Uni-dimensional Variation Tool* (1dVT), that is number-line-like sliders, each corresponding to one of the parameters used in a Logo procedure, will have at first to deconstruct the given construction, with the aim to detect how the parameters function in the composition of the object presented in the Scene. In addition to this, in the given notation code all types of angular relationships are found. These are some difficulties that the pupils have to encounter and are likely not to distinguish their role in the cone construction while tinkering with the half-baked microworld. By means of the "dragging" tool the students also perceive that the figure they have constructed is a "snapshot" of a broader class of figures with some common characteristics. The students eventually, with the combined use of visual and notational representation along with the functionalities of the bi-dimensional variation tool of parameters (:a) and (:b) are expected to be able to ascertain their linear dependence (they end up in relating the length (:b) to the slant angle of the turtle through the mathematical formula :b= :a) (Table 1, column 2 presents a possible version of the code of the cone)

The third goal involves a more "refined" method of tinkering with the half-baked microworld that is the attempt to mathematize further the behavior of the turtle in the construction of the cone. The use of the variation tool *Bi-dimensional Variation Tool (* 2dVT) leads to the expression of further abstraction, according to its structure and use. Intuition dictates that to approximate as best as possible the cone, the number of iterative constructions of its generatrices must increase, namely their *in between distance has to be more dense*. How can they generalize the formula in the notation code, so that the turtle won't turn around a generalized angle theta = :x ? And then what is the number (:n) of generatrices the has turtle has to create, to form always a closed polygonal path so as to approximate as closely as possible the circumference of a circle? Articulating relevant hypotheses conjectures the students will be able to investigate them by means of the bi-dimensional variation tool. The graphical representation of pairs of values of these parameters ensuring that the turtle's motion is always as circular as possible will lead to a hyperbola (Figure 2 depicts the relationship between parameters :x and :n)

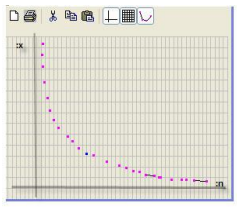

Figure 2. Relationship between (:x) and (:n) parameters

Epilogue

This teaching proposal has not yet been implemented in practice in order to present some

pragmatic results. Some of the empirical mathematics related to construction that comes forth through the manipulation of the microworld "half_baked" depending on the chosen course of action is: Students investigate the relationship between the number of degrees of an angle and the length of the generatrix as a necessary condition to create the cone. This concept is the core around which they are invited to create a number of meanings using the symbolic (the programming code), graphical (the visual representation) and the variation tools of the 3D microworld. The students through the investigation of the half-baked microworld, while tinkering with it, manage to transform it into a complete program that permanently constructs a cone, seize the rules behind the process of its construction. The notion of angle as change of direction and plane, the discreteness and the continuity of the number that is indicated implicitly through the alteration of the step of the slider in the 1dVt variation tool, the inverse proportionally quantities, the central angle compared to the length of chord from which the corresponding arc is shown, the investigation of angular relationships are some of the mathematical meanings that the students create while trying to debug and improve the program.

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