Supersonicman – an informatics x physics project

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Abstract

In the paper, a student project is proposed where high altitude fall of a person in the air is investigated. The object is to answer the question if it is possible to reach supersonic speed.

Keywords

Constructivism, spreadsheet, model, Kittinger, fall

Introduction

In the paper, a project for grammar school student (age 17-19) is proposed where high altitude fall of a person in the air is investigated. The object is to answer the question if it is possible to reach supersonic speed. An interactive numeric model of J. W. Kittinger's legendary jump is created in Excel, based on the Euler's method which is clear and intelligible. No programming is used. Using the model, students can investigate the behaviour of the system and find the boundary or limiting cases. The project meets well the UNESCO's notion about ICT in secondary school (Anderson, 2002). It corresponds with 3 out of the 11 units of the module "Application of ICT in subject areas", which are: ICT in Natural Sciences, Modelling and Simulation, and Spreadsheet Design. The project is based on author's article (Benacka, 2011). Ideas for other cross subject projects (informatics x physics, informatics x mathematics) can be found in (Benacka, 2007, 2008, 2009, 2011b). The minimal ICT tools that the students will use in the project are: Internet to find the sources; word processor to write up the report; PowerPoint, to prepare the presentations, and spreadsheets as the key tool to develop the model. The choice of the spreadsheet is obvious – it is a widespread program that enables students to analyse scientific problems and find solutions without programming. Not only is it easy to use, but it allows using problem-solving and heuristic methods, which are close to talented pupils. While creating the model, students practise their spreadsheet skills, gain new ones, and get a better understanding of the modelled problem. This makes spreadsheet an excellent tool for constructivist learning.

High-altitude fall in the air

On August 16, 1960, USAF Captain (later Colonel) Joseph W. Kittinger carried out his legendary jump from the helium balloon Excelsior III at the altitude of 31,300 m. He reached the top speed of 274 m/s, which was 0.9 of the speed of sound at the altitude. His mass was 142 kg, from which 70 kg was gear. He fell as sitting in an armchair due to his inflated pressure suit. He had serious breathing difficulties between 27,400 m and 21,300 m due to the helmet that was pressing against his throat (Kittinger, 1960; URL 1). It has been the highest, longest, and fastest sky-dive ever made. According to some sources, the top speed was 319 m/s (Clash, 2003). That would be a supersonic fall at the altitude. Some time ago, an attempt was cancelled to break the sound barrier in fall (Tierney, 2010). The jumper was to fall in a special suit, head-to-earth, stabilized just with his legs and arms straighten back in a "V" shape. There is a question: Is it possible for a person falling in the air to reach supersonic speed, that is, to become a Supersonicman?

US Standard Atmosphere is a scientific atmosphere model (URL 2). The properties are in Tab. 1.

Layer b	Altitude (km) $z_b - z_{b+1}$	Density ρ_b (kg/m^3)	Temperature T_b (K)	Temperature lapse rate L_b (K/m)	Speed of sound c_b (m/s)	Name
0	0 – 11	1.225	288.15	–0.0065	340.29	Troposphere
1	11 – 20	0.36391	216.65	0	295.07	Stratosphere
2	20 – 32	0.08803	216.65	0.001	295.07	

Table 1 US Standard Atmosphere 1976 up to 32 km

Values ρ_b, T_b, and c_b hold at bottom z_b of layer b. Temperature lapse rate L_b is constant within layer b. In layers $b = 0$ and $b = 2$, density $\rho(z)$ is given by the equation

$$\rho = \rho_b \left[1 + L_b \left(z - z_b \right)/T_b \right]^{-(\beta/L_b + 1)}, \tag{1}$$

where $\beta = g_0 M / R$, $R = 8.31432\,\text{J/(mol.K)}$ is the gas constant, $M = 28964.61\,\text{kg/mol}$ is the molar mass of the air, and z is the altitude, where $z_b \leq z \leq z_{b+1}$. Remark: In layer $b = 1$, which is out of the interest, the density is $\rho = \rho_b\, e^{-\beta(z-z_b)/T_b}$. Speed of sound $c(z)$ is given by the equation

$$c = c_b \sqrt{1 + L_b \left(z - z_b \right)/T_b}. \tag{2}$$

Acceleration due to gravity at sea level is $g_0 = 9.80665$ m/s^2. The acceleration at altitude z is

$$g = g_0\, r_0^2 / \left(r_0 + z \right)^2, \tag{3}$$

where $r_0 = 6{,}356{,}766$ m is the effective radius of the Earth.

Weight $G = mg$ and drag $F_D = 0.5 C A \rho v^2$ (Marion, 1970) act on a body of mass m falling in the air, where A is the maximum cross-section area of the body perpendicular to the motion direction, ρ is the air density, v is the speed, and C is the drag coefficient dependent on the shape of the body. If the speed is subsonic (below about 0.8 of sonic speed = Mach 0.8), then C is virtually constant. If the speed is transonic (from Mach 0.8 to 1.2), then C increases rapidly. The resulting force F is

$$F = G - F_D. \tag{4}$$

It holds that $F = ma$, where $a = \Delta v / \Delta t$ is acceleration and t is time. Substituting in Eq. (4) gives

$$\Delta v = \left(g - 0.5 S C \rho v^2 / m \right) \Delta t, \tag{5}$$

where $\Delta t = t_{\max} / n$, and n is the number of subintervals of interval $[0, t_{\max}]$. It holds that $\Delta z = v \Delta t$. It holds at $t = 0$ s that $v = 0$ m/s and $z = h$. Then, speed $v(t)$ and altitude $z(t)$ are given by the equations

$$v_i = v_{i-1} + \left(g - 0.5 S C \rho v^2 / m \right) \Delta t, \ v_0 = 0, \ i = 1, \Lambda, n, \tag{6}$$

$$z_i = z_{i-1} - v \Delta t, \ z_0 = h, \ i = 1, \Lambda, n. \tag{7}$$

Equations (6) and (7) allow graphing the speed and altitude.

Implementing the model in Excel

The application is in Fig. 1. The graph is made over 5,000 points. The white cells are for inputs.
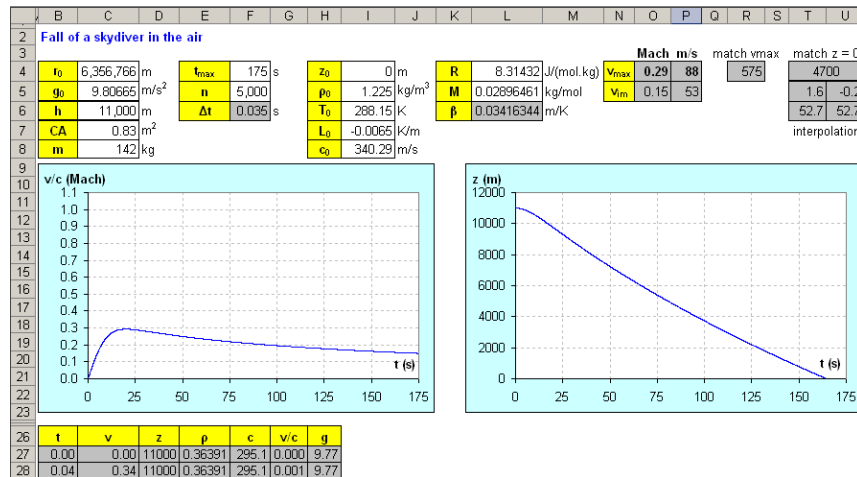

Figure 1. Speed and altitude of a person falling in the troposphere:
$h = 11,000\ m,\ CA = 0.83\ m^2,\ m = 142\ kg,\ v_{max} = 88\ m/s = Mach\ 0.29$

Instead of C and A, product CA is inputted (see the next section). The grey cells contain formulas. In cell F6, it is =F4/F5. In L6, it is =C5*L5/L4. The model is in cells B27:H5027. They contain the following formulas (copied down as far as row 5027; the number of the equation is added):
B27 =0; C27 =0; D27 =C6
E27 =$I$5*(1+$I$7*(D27-$I$4)/$I$6)^(-$L$6/$I$7-1) (1); F27=$I$8*SQRT(1+$I$7*(D27–$I$4)/$I$6) (2)
G27=C27/F27; H27 =$C$5*($C$4/($C$4+D27))^2 (3);
B28 =B27+$F$6; C28 =C27+(H27-0.5*$C$7/$C$8*E27*C27*C27)*$F$6 (6); D28 =D27–C28*$F$6 (7)

The maximum of relative speed v/c is in cell O4 found by function MAX(G27:G5027). In cell R4, function MATCH(O4;G27:G5027;0) returns the ordinal number of the maximum. In cell P4, function OFFSET(C26;R4;0;1;1) returns the value in the cell shifted from C26 downwards by the number in cell R4, i.e., it returns the speed from the row where the maximum relative speed is.

The impact speed is calculated in cells T4:U6. In cell T4, function MATCH(0;D27:D5027;–1) gives the ordinal number of the null or last positive altitude (from D27 downwards). In cell T5, function OFFSET(D26;T4;0;1;1) gives the altitude. The next altitude is returned into cell U5 by function OFFSET(D26;T4+1;0;1;1). Thus, cells T5 and U5 contain the last nonnegative and the first negative altitudes. In cells T6 and U6, functions OFFSET(C26;T4;0;1;1) and OFFSET(C26;T4+1;0;1;1) give the speed from these rows. The impact speed, which is the speed at null altitude, is calculated in cell P5 using linear interpolation by the formula =T6–(T5–0)/(T5–U5)*(T6–U6). There is no sense to calculate the impact speed in the stratosphere (Figs. 2, 3).

Analysis of the fall

The application with the data for Kittinger's jump is in Fig. 1. Parameter CA was iterated until the maximum speed was 274 m/s, which gave $CA = 0.83\ m^2$. It holds for a person that $C \sim 1-1.3$ (URL 3). Kittinger fell as sitting in an armchair with load on his back. If $C \sim 1$, then $A \sim 0.83\ m^2$, which is acceptable Kittinger was in transonic range (above Mach 0.8) for 28 s from $t = 29$ s to $t = 57$ s. If C increased during this flight to $C \sim 1.3$, then $A \sim 0.64\ m^2$, which is still acceptable. This version of the fall is plausible.
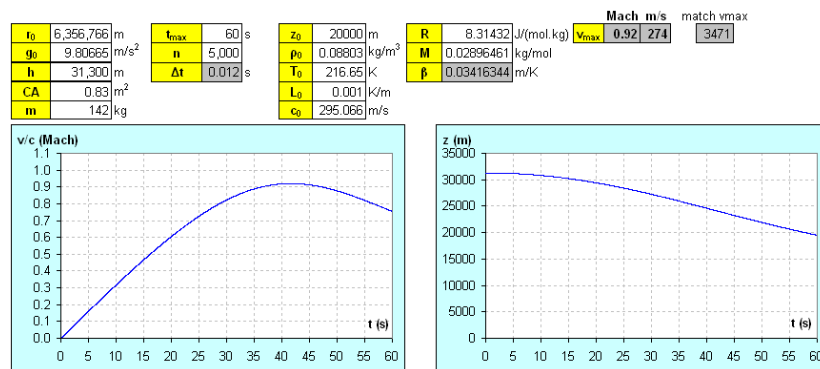
r_0	6,356,766	m	t_{max}	60	s	z_0	20000	m	R	8.31432	J/(mol.kg)	v_{max}	0.92	274	match vmax 3471
g_0	9.80665	m/s²	n	5,000		ρ_0	0.08803	kg/m³	M	0.02896461	kg/mol				
h	31,300	m	Δt	0.012	s	T_0	216.65	K	β	0.03416344	m/K				
CA	0.83	m²				L_0	0.001	K/m							
m	142	kg				c_0	295.066	m/s							

Figure 2. Speed and altitude of a person falling in the stratosphere:
$h = 31{,}300\ m,\ CA = 0.83\ m^2,\ m = 142\ kg,\ v_{max} = 274\ m/s = Mach\ 0.92$

It is clear from Fig. 3a that reaching maximum speed of 319 m/s (Mach 1.08) is possible if $CA = 0.46\ \text{m}^2$. If $C \sim 1$, then $A = 0.46\ \text{m}^2$, if $C \sim 1.2$, then $A = 0.38\ \text{m}^2$ and if $C \sim 1.3$, then $A = 0.35\ \text{m}^2$. The values of A are too small. It is impossible that Kittinger could reach this speed.
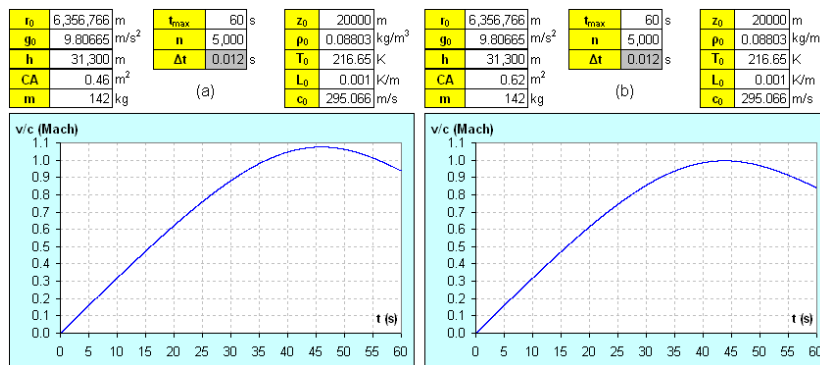

Figure 3. Speed and altitude of a person falling in the stratosphere: h = 31,300 m, m = 142 kg,
(a) $CA = 0.46\ m^2$, $v_{max} = 319\ m/s = Mach\ 1.08$, (b) $CA = 0.62\ m^2$, $v_{max} = 296\ m/s = Mach\ 1$

Fig. 3b shows that sonic speed (Mach 1) could only be reached if $CA = 0.62\ \text{m}^2$. If $C \sim 1$, then $A = 0.62\ \text{m}^2$, if $C \sim 1.2$, then $A = 0.52\ \text{m}^2$ and if $C \sim 1.3$, then $A = 0.48\ \text{m}^2$. Also these values of A are too small for the Kittinger's way of fall.

Suppose Kittinger would fall head-to-earth, arms and legs straiten back, and with well-shaped load. It holds that $C < 1$ for such a system at subsonic speed, so $C \sim 1$ is possible at sonic speed. The corresponding area $A = 0.62\ \text{m}^2$ is acceptable (Fig. 3b). If $m = 100$ kg, then the model gives $CA = 0.43\ \text{m}^2$; if $C \sim 1$ then $A = 0.43\ \text{m}^2$, which is still acceptable. Thus, reaching sonic speed is possible. The question is whether the jumper would survive. Kittinger had serious breathing difficulties from 27,400 m to 21,300 m because of the helmet that was pressing against his throat. Fig. 2, right side, shows that it was from $t = 29$ s to 57 s. Fig. 2, left side, shows that Kittinger was just in the transonic range, that is, above Mach 0.8. Then, "Parts of your body may be going supersonic while others aren't, causing flutter waves pulling back and forth … that knocks him out of control" (Tierney, 2010). This turbulence caused tragic plane crashes when breaking the sound barrier at the end of forties. The problems with the helmet could not have been caused by anything else.

Fig. 1 shows a hypothetical Kittinger's fall from 11 km where the troposphere ends. Passenger airplanes cruise at this altitude. The maximum speed is 88 m/s, which is just Mach 0.29. To reach 100 m/s, it has to hold that $CA = 0.62\ \text{m}^2$. If $m = 100$ kg, then the model gives $CA = 0.44\ \text{m}^2$.

Conclusion

The project shows the great possibilities that spreadsheet offers for studying school subjects. The facts in section 2 are additional to physics curriculum, Eqs. (1) – (3) show using higher functions in practice. Calculating the impact speed is an example of getting a value that is not in the cells by interpolation using the values returned by functions MATCH a OFFSET. They are important for those who will use Excel for modelling in science, engineering, business, etc. The analysis is an example of scientific argumentation to find the solution. The result is: A person falling in the air can reach supersonic speed if he falls in the stratosphere from the altitude of about 32 km head-to-earth. Surviving is doubtful. In the troposphere, the maximum speed is about 100 m/s.

Acknowledgement

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