Building the System of Designing Own Mathematics Textbook

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Abstract

Students in South Korea receive standardized education as learning contents and order of contents in mathematics textbooks follow the national curriculum. In order to find the learning order or methods best suited to learners, therefore, it is necessary to analyze the contents in the textbooks by elements and reconstruct them. From a constructionism perspective, this research will first analyze mathematics textbooks in South Korea to build an environment where teachers or students can design their own textbook by dividing the contents into pieces, and then look into the principles of web-based system design.

Keywords

Constructionism, Mathematics Education, Learning Object, National Curriculum

Introduction

Pointing out the problem of traditional mathematics textbooks which deliver contents in a given order, Freudenthal (1973) emphasized the need to reconstruct textbooks so that it enables students to experience 'guided reinvention'. Recent studies which stress the historic-genetic approach confirmed order of contents in the traditional textbooks may not always be appropriate to all students. Some teachers may prefer exploration-based learning while the others may prefer historic-genetic learning. However, one standardized textbook cannot support such different classes; therefore, teachers should be able to reconstruct the textbook on their own according to the learning objectives and the way to deliver the contents.

However, reconstructing mathematics textbooks in reality is quite challenging in some countries. For example, South Korea has a national curriculum and the national institution named Ministry of Education, Science and Technology (MEST) proclaims the standardized national curriculum. Like other subjects, mathematics textbooks are designed in accordance with the national curriculum which has been revised approximately every five year. All of elementary school students in South Korea use one single textbook and learn mathematics from the same contents in the same order. In middle schools and high schools, teachers choose one of the several government authorized-textbooks and all of the students in one school learn mathematics from the same textbook chosen. Being able to make a choice may sound better, but order of contents is almost the same although learning materials can be a bit different because all textbooks follow the national curriculum. That is, any math teachers in South Korea teach students mathematics in the same teaching order.

We anticipate developing technology would enable such traditional textbooks to be more flexible. Especially, we expect to build an environment where teachers and students can experience designing their own mathematics textbook. In order to reconstruct Korea's mathematics textbooks, the objective of this research is to look into the designing principles of the environment where teachers or students can analyze contents of textbooks by elements, draw units, and redesign their own textbook by putting the units together.

Theoretical Perspective

Ackermann (2004) argued that in comparison of Papert's constructionism and Piaget's constructivism, although both theories are of the same view, the roles of the media are more emphasized in constructionism. Here, the media being emphasized is a physical construction environment for mental construction. Likewise, Kafai and Resnick (1996) said that the core of constructionism is the mental construction through physical construction.

> Constructionism is both a theory of learning and a strategy for education. It builds on the "constructivist" theories of Jean Piaget, asserting that knowledge is not simply transmitted from teacher to student, but actively constructed by mind of the learner. Children don't get ideas; they make ideas. Moreover, constructionism suggests that learners are particularly likely to make new ideas when they are actively engaged in making some type of external artifact-be it a robot, a poem, a sand castle, or a computer program-which they can reflect upon and share with others. Thus, constructionism involves two inter-wined types of construction: the construction of knowledge in the context of building personally meaningful artifacts (Kafai and Resnick, 1996).

The computer environment to implement constructionism must be the one where learners can construct the artifacts they want and construct knowledge through the very construction activity. In other words, through this learning by making, learners naturally come to know of powerful ideas through the activity of physical construction.

Is internet an appropriate space for constructionism? Resnick (1996) used the term, 'distributed constructionism' as discussing how technology like internet can contribute to constructionism. He explained internet can help construction activities in the following three ways.

- Discussing constructions
- Sharing constructions
- Collaborating constructions

Internet here is a tool to discuss and collaborate construction, rather than a tool for construction. In fact, the internet web in the early stage was the media that deliver information existing in a virtual space to users. Therefore, researches on web-based learning generally aimed at resolving spatial or physical limitations through technology, and delivering the existing knowledge in a more effective way. Recently, however, web has been developed to an environment that can realize user-generated contents with interactive technology advancement. Web-based learning researches have expanded from studies on virtual space design that simply promotes knowledge delivery to studies on an environment which allows individually and socially meaningful physical construction. In other words, web has been developed from an instructionism tool for knowledge delivery to a constructionism tool for knowledge construction.

Going even further than just constructing a virtual object, web can be used as a space where people make their own hamburger or book (http://pediapress.com) that they can actually get hold of. Hamburger 2.0 (http://www.4food.com/) is an environment where people make their own unique hamburger by choosing a type of bread and other ingredients prepared and recombining them in the order that they like.

Churchill (2007) defined ‘learning object’ as a thing designed, as considering reusability, to be used in a different form of learning, utilizing various media. Learning objects are sort of learning resources and they may provide learners with a variety of customized education environments which help learners achieve the learning objectives depending on the way to combine the resources. Here, we would like to emphasize on reusability of learning unit elements.

From a constructionism perspective, we would like to design an environment where mathematics textbooks can be reconstructed. This environment is a web space which enables teachers or learners to reconstruct mathematics textbooks by adjusting order of learning objects and show learning contents provided by the national curriculum as a combination of various learning objects.

Analysis of Korean Mathematics Textbooks

According to South Korea’s national curriculum revised in 2009, all students up to 3rd grade of middle school are supposed to complete the same curriculum (MEST, 2009). One distinctive feature in 2009 revision is that it did not rule order of learning while setting the required learning contents. The mathematics curriculum was revised in 2011 based on this principle and Table 1 shows the number of learning contents given in respective grade’s curriculum (MEST, 2011). That is, studying the learning contents shown in Table 1 is mandatory to all elementary and middle school students.

Elementary 1st ~ 2nd grade	Elementary 3rd ~ 4th grade	Elementary 5th ~ 6th grade	Middle 1st ~ 3rd grade
14	24	20	29

Table 1. The number of achievement standards in Required Courses

High school students are supposed to complete a few subjects chosen from 6 electives, depending on their future career plan. Even if these are electives, the learning contents in each elective are decided by the government. Table 2 shows the number of learning contents in each elective.

Mathematics I	Mathematics II	Calculus I	Calculus II	Probability and Statistics	Geometry and Vectors
12	9	10	8	8	8

Table 2. The number of achievement standards in Electives

Newly developed mathematics textbooks have similar order of learning because the contents are similar to the ones covered in the pre-revision textbooks.

We would like to design an environment where we can reconstruct mathematics textbooks by rearranging order of contents while not getting out of the learning contents given in the national curriculum. However, this does not mean simply changing order of contents. A learning content is comprised of several learning objects and we need to make all learning objects' order changeable as an independent unit. For learning materials, in addition, we would provide various learning objects that allow construction of various learning contents with one learning objective in order for learners to design unconstrained learning path. And we would unify the form of learning objects given in learning contents. Having a variety of learning objects is to allow teachers or students to make various combinations and unifying learning objects' form is not to be bound by grade when creating mathematics textbooks.

The form commonly presented in several textbooks can become an appropriate form of learning objects. For this, we analyzed textbooks used in elementary schools and in the 1st, 2nd and 3rd grade of middle schools. Table 3 shows the number of textbooks analyzed. We particularly focused on analyzing the middle school textbooks and this is because the design conducted in this research plans to be built in 2012 and applied to middle school students on a trial basis.

Elementary School	Middle School – 1st grade	Middle School – 2nd grade	Middle School – 3rd grade
1	27	17	14

Table 3. The number of mathematics textbooks

Designing the System

As a result of textbook analysis, we were able to find out many learning contents generally have the same learning order. In addition, we concluded a learning content has the following 7 types of learning objects in common although it slightly varies depending on the textbooks. Also we learned order of learning objects are presented in an identical order in most of the textbooks. The followings are the 7 types of learning objects that we discovered.

- Conceptual learning: This is about concepts and explaining the concepts. The concept explained here can be more than one. It does go beyond what traditional mathematics textbooks covered.
- Exercise problem: This is about problems to explain a certain concept or simple problems that apply the concept understood. Mathematics textbooks in South Korea include a variety of easy to difficult problems. Textbooks present the problems when one part of learning contents is completed or use the problems to explain a certain concept.
- History of mathematics: Many mathematics education researches stress using history of mathematics in the teaching (Freudenthal, 1983). History of mathematics can arouse students' interest or can be used as a tool to develop the contents according to the order of historical occurrence of mathematics. Such endeavours have been included in mathematics textbooks since 1990's. In most cases, this is limited to listing up historical facts on mathematics in text; however, this can be presented in a dynamic form such as a video clip or animation if we can use a web-based virtual space.

- Exploration: Mathematics also requires exploration as other subjects in natural science do. A strategy like 'What if not' is a very important strategy in mathematical thinking and problem solving. For this, it is necessary to have mathematics laboratory where students can explore and experiment their conjecture. Even if current mathematics textbooks cover it in the form of 'Learn More about It', paper textbooks reach the limit to expressing dynamic mathematical experiments. However a web-based space can make dynamic mathematical experiments possible. In this context, introducing Microworld and mathematical experiment using the environment are meaningful. Edwards (1995) defined the microworld as follows and explained about exploratory learning, using microworld.
  - A microworld contains a set of computational objects.
  - A microworld links more than one representation of the underlying mathematical or scientific entities or objects.
  - Often the objects and operations in a microworld can be combined to for more complex objects or operation.
  - Typically, a microworld includes a set of activities (Edwards, 1995).

  It is desirable to provide Logo (Abelson and diSessa, 1980; Papert, 1980), Geogebra (http://www.geogebra.org), JavaMAL (Cho et al., 2010) in learning object and it is also feasible in reality.
- Mathematical communication: The mathematics curriculum revised in 2009 (MEST, 2011) emphasizes on mathematical communication in addition to mathematical reasoning and mathematical problem solving. With the stream, recent textbooks include open-ended mathematics questions that encourage students to discuss to answer. If these contents are linked to Social Networking Service (SNS) technology, mathematical communication in online spaces would become available. Then, we may expect active communication environments where students and teachers ask questions and answer thru SNS like Facebook or Twitter. This can be a good way to actualize distributed constructionism that Resnick (1996) mentioned.
- Problem solving: Many countries have emphasized on problem solving ever since Pólya (1957) addressed one of the most important objectives in mathematics education is problem solving. Besides NCTM (2000), problem solving is one of the critical objectives in South Korea's mathematics curriculum. Most of the mathematics textbooks in South Korea include problem solving parts separately in the middle or end of chapters. The problems presented in this learning object are different from the exercise problems presented to help students understand a certain concept. They are rather related to a real life or include mathematics problems and experiments with solutions which require many stages of thought process.
- Cartoon: Mathematics textbooks that can be somewhat dry may also include interesting contents. In a web space, these contents can be provided in the form of webtoon or animation unlike cartoons in a paper textbook. If contents are constructed with well-structured characters in the whole context and frame, this may stimulate learners' interest and achieve educative results to some degree.

The 7 types of learning objects reviewed above may have various representations and they can be represented in the form of text, video or even manipulable laboratory.

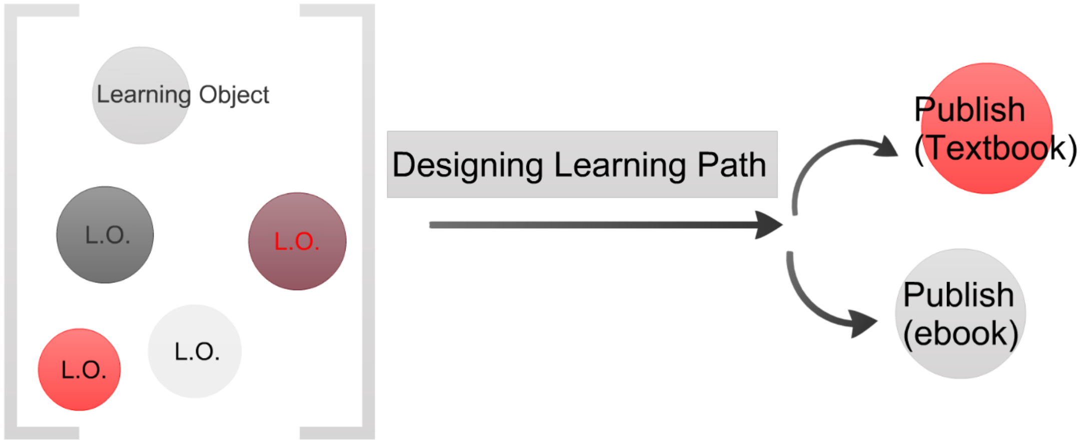

Figure 1. Designing own mathematics textbook

A Prototype of the System

Figure 1 is the image of the system that we design. The learning objects in Figure 1 are from the 7 types of learning objects reviewed above. Users may publish their own book by combining different learning objects and this textbook can end up with a paper book or an e-book with dynamic presentation. Some may only insert video-clips in their own textbook while leaving table of contents as it is. Others may create a paper comic book or an e-comic book by adding different cartoons. Students and teachers would learn by creating a physical construction - their own mathematics textbook, and learn mathematics by using such physical constructions.

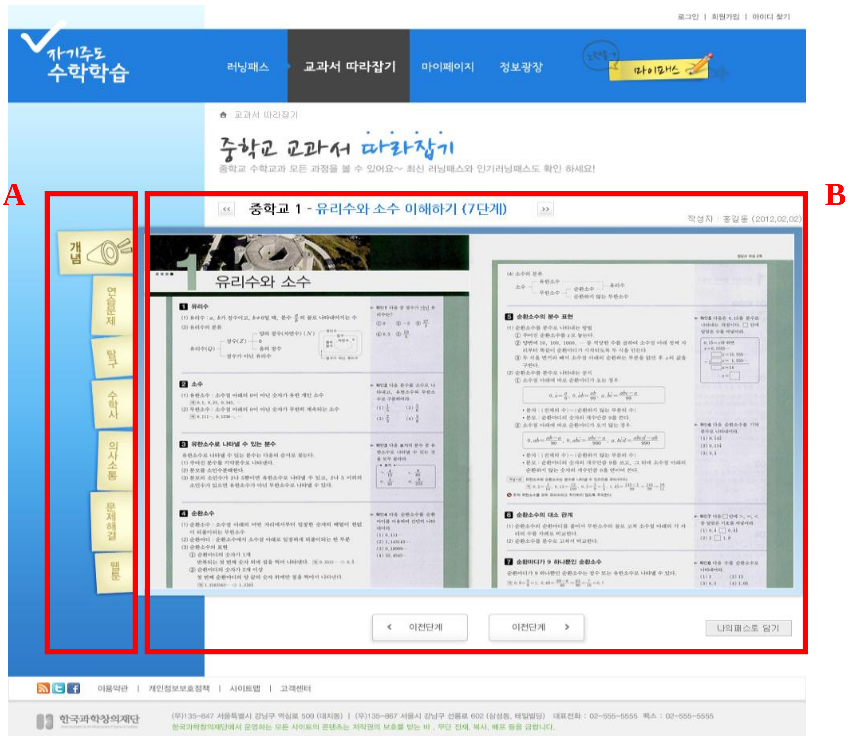

Figure 2. Prototype of mathematics learning website

Figure 2 is the image of the system represented in a web space we designed. Part A shows the learning objects included in learning contents and Part B shows the items in learning object. Figure 2 shows 'Conceptual learning' learning object in the 'Rational Number and Prime Number' learning content. By clicking the learning objects in sequence presented in Part A, users may study the contents as intended by the content designer. Or they may study the contents on their own way by clicking the learning objects that they are interested. In addition, users can refer to the curriculum that covers the learning object that they are studying regardless of which one it is because all learning contents are included in South Korea's curriculum. This is very important as teachers and students in South Korea are very familiar with content organization and content order in textbooks due to the unique government control on the curriculum and textbooks.

Figure 3. Prototype of creating mathematics textbook

Figure 3 is a tool that enables teachers or students to create their own mathematics textbook. Users can choose learning contents in STEP 1. Teachers and students in South Korea are familiar with the way contents are organized in textbooks; therefore providing order of contents is convenient when they look for certain contents. They can certainly find the contents by searching keywords as well.

Learning objects are presented in STEP 2 once the contents that users would like to include are dragged from STEP 1. In STEP 2, users can pick and choose certain learning objects and put them in their My Path. Users can choose up to 7 learning objects and also select one learning material from respective learning objects as learning materials are pre-registered in each learning object. For example, there can be three cartoons related to negative number while there can be one cartoon related to fraction. Usable learning objects are added continuously. Teachers and students put all learning objects that they would like to include into My Path by coming back and forth between STEP 1 and STEP 2. In STEP 3, they complete to create their own mathematics textbook by adjusting order of learning contents in My Path and putting a title. The completed mathematics textbooks can be used by other students and teachers in online spaces or can be delivered to the users after the bookbinding process.

Closing Remark

This research examined South Korea's mathematics curriculum and textbooks. In South Korea, the government is responsible for curriculum research and development, and allows only government-authorized textbooks to be used in schools. As analyzing textbooks, we found out mathematics textbooks include learning contents in common in a given order, and each learning content is comprised of 7 types of learning objects. Based on this finding, we looked into the features of the web-based system and the prototype where the forms of constructionism-based mathematics textbook can be realized. Then, we built the system and the prototype where users can create their own mathematics textbook, using modulized learning objects selectively, given that the contents in mathematics textbooks have a certain forms (learning contents and learning objects).

This research will be followed by another study, not just limiting it to a theoretical study. The system we designed has a plan to be built in the second half of year 2012, targeting middle school students. Once the service begins, we will follow up with a study to see how teachers and students use the system in their real life. In particular, we will intensively look into how and in what forms the new type of textbooks used in a web space would be constructed.

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