Gestures as a tool of semiotic mediation in 3d turtle geometry environment

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Abstract

In this paper we report findings from a design-based research aiming at shedding light on the way eleven-year old students used gestures while constructing mathematical meanings in the framework of a 3d Turtle Geometry Environment. The results bring in the foreground the role of gestures, as signs that mediated the mathematical notions integrated in the computational environment, as well as the interconnection between gestures and the embodied turtle metaphor. Different kinds of gestures were used by students depending on their focus point during the construction processes. When focusing on turtle's navigation and the graphical results of this navigation, students used dynamic representational gestures. On the contrary when viewing 3d space or 3d objects as external observers they used abstract deictic or static representational gestures.

Keywords (style: Keywords)

gestures, turtle geometry, embodied metaphors

Theoretical Background

The study of gestures is a rather new field of research in mathematics education (Radford, 2009), which is investigated through various perspectives. In this paper the focus is set on the way students at the end of primary school used gestures in the framework of constructionist activities in a 3d Turtle Geometry computational environment. Gestures are investigated as a special mode of embodied expression and communication and as a tool of semiotic mediation of the learning process, which -in conjunction with other modes of expression- can shed light on the processes of mathematical meanings construction (Arzarello et al., 2009).

According to the socio-cultural theory of learning people's interaction with the real world is defined and formed through the use of symbolic objects and cultural tools (Vygotsky, 1978). In this framework gestures are investigated in their semiotic perspective, on the way they function as signs that mediate people's interaction with their environment in specific cultural contexts (Radford, 2005). In particular, in mathematics education gestures seem to acquire new dimensions (McNeil, 2000) and are thought as a means of knowledge objectification, as a means that can help students realise the notions integrated in the various mathematical objects (Radford, 2009). A special interest is aroused in cases that digital technologies are used, where certain actions are conceived as new kinds of gestures, e.g. pointing with the mouse or 'dragging' of hot spots in Dynamic Geometry computational environments (Kaput, 2005). On the one hand gestures are related to the task to be accomplished and on the other hand they may be related to the mathematical knowledge that is to be attained. Gestures are usually contingent to the situation

determined by the solution of a particular task but they can also play a rather pivotal role in promoting the evolution of signs from idiosyncratic to culturally determined and crystallised mathematical signs. The relationship between digital technologies and knowledge is complex (Bartolini Bussi & Mariotti, 2008) and a careful analysis of the evolution of gestures used by students (among all the other signs) can offer a new perspective on the access that students have on the embedded in these artefacts mathematical knowledge.

In parallel, the intimate relationship between the functioning of the brain and body experience (with or without the use of tools) even when the most abstract mathematical notions are considered is now commonly recognised. Concepts are imminent in each concrete realisation of experience and in its relation to other experiences. Nemirovsky (2003) argues that 'thinking is not a process that takes place 'behind' or 'underneath' bodily activity, but it is the bodily activities themselves'. Thus, gestures are investigated as a window on the embodied aspects of meaning construction processes (Anastopoulou et al., 2011), as an interface between abstract and symbolic mathematics and mathematical metaphors (Kim, Roth and Thom, 2010) that are on their part grounded on human sensorimotor experience and action (Lakoff & Nunez, 2000). Research interest has also arisen recently in relation to the role of gestures while students are constructing geometrical figures in 2d (Latsi, 2010) and 3d Turtle Geometry (Morgan & Alshwaikh, 2009) computational environments. These studies provided empirical support for the embodied means used by students in their effort to carry out particular geometric tasks highlighting the connections of certain gestures with the integrated in the aforementioned digital tools mathematical knowledge while raising questions about the interpretation and use of the same gestures by different groups of interlocutors.

In the research presented here our pedagogical aim was to engage the students in navigating a moving entity, the turtle, to construct graphical digital objects through Logo programming. Research seems to conclude that carefully designed 2d Logo- based microworlds are an effective medium in offering rich mathematical experiences and encouraging the construction of meaning through the turtle metaphor (Clements & Sarama, 1997; Kynigos, 1997). Navigating the turtle requires the formation of essentially novel methods of spatial orientation, where the reference point is not the position of the user's body but the turtle's body, relative to which the entire system of orientation may change. In this framework body-syntonicity is a critical concept in 2d Turtle Geometry (Papert, 1980) that refers: a) to navigating the turtle by coordinating one's body-posture, physically or imaginary, with the turtle-vehicle of motion and b) to solving geometrical problems drawing upon ones embodied motional experiences. Recent extensions of Turtle Geometry in 3d space do not offer just a new perspective in the teaching and learning of geometry. New issues are raised related to the way the turtle metaphor is put to use and the way deeply rooted intuitions about experiencing space and locomotion can be exploited so as to make sense of geometric notions (Kynigos & Latsi, 2007, Morgan et Alshwaikh, 2009). In particular our research aim was to investigate: a) students' gestures and their role in mathematical meanings construction in the 3d simulated geometrical space and b) the way these gestures are related to the central metaphor in the 3d Turtle Geometry environments, turtle as a moving entity with which the user can syntonise his/her body.

The computational Environment

MaLT is a constructionist microworld environment that extends 'Turtleworlds' to 3d geometrical space. 'Turtleworlds' blends Logo based Turtle Geometry with tools to dynamically manipulate procedure variables and observe the resulting 'continuous' change to the respective figural constructions (Kynigos et al, 1997). In MaLT, we used a well established method to extend Turtle

Geometry to 3d by adding two kinds of turn commands (Reggini, 1985): 'UPPITCH/DOWNPITCH n degrees' ('up/dp n'), which pitches the turtle's nose up and down on a plane perpendicular to the one defined by right-left turns, and 'LEFTROLL/RIGHTROLL n degrees' ('lr/rr n') which moves the turtle around its own axis. A second feature of MaLT is that we kept the 'Turtleworlds' feature of variation tools. These tools recognise the procedure responsible for any figural construction and afford dynamic manipulation of variable values resulting in DGS-style change in the figures. A third feature also affords dynamic manipulation but this time what is changed is the users' viewpoint of the Turtle Geometry space: a) in a toggle fashion by using buttons to pick among 3 default views (front, side, top-down) and b) by dragging a specially designed vector tool, which we called 'the active vector', where the user can define the camera's direction or position. Thus MaLT combines: a) interactivity, b) multiple interlinked representations and c) dynamic manipulation and dynamic visualisation of the 3d simulated space.
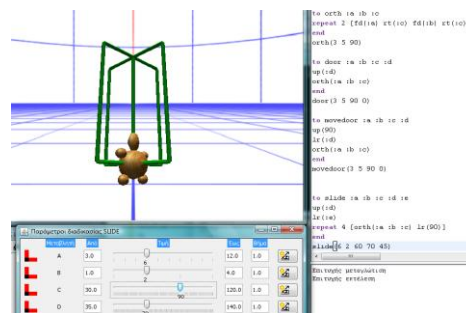

Figure 1 .The MaLT computational environment

Methodology

Espousing an interpretive approach in educational research (Cohen et al., 2007) in the study reported here we followed a design-based research method (Van Den Akker et al., 2006), which entailed the 'engineering' of tools and task, as well as the systematic study of both the process of learning and the means of supporting it (Gravemeijer & Cobb, 2006). A critical component of design –based research is that the design is conceived not just to meet local needs but to advance a theoretical agenda, to uncover, explore and confirm theoretical relationships, to create new theoretically expressed understandings about areas for which little is known. Thus, the analysis we have carried out does not comprise any kind of quantification of qualitative data, but rather refers to a non mathematical process of interpretation, carried out for the purpose of discovering concepts and relationships in raw data and then organizing these into a theoretical explanatory scheme.

The research took place in the 6th grade of a public primary school in Greece. The class consisted of 23 pupils, who had totally sixteen 45 minutes teaching sessions with the experimenting teacher over two months. The pupils didn't have any previous experience with 3d Turtle Geometry environments but they were accustomed to 2d Turtle Geometry. The pupils worked collaboratively in mixed-gender groups of two or three in the school's computer laboratory. The tasks were designed to bring in the foreground issues concerning the mathematical nature of 3d geometrical objects through their dynamic manipulation and transformation in mathematically meaningful ways. In this research paper we present and analyse data taken from the first two tasks of the activity sequence that lasted 4 teaching sessions. In the 1st task the pupils were asked to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft

while in the second one they were asked to construct rectangles and to position them in at least two different planes of the simulated 3d space, so as to simulate the walls of a room.

In order to describe the pupils' learning trajectories as they happened in real time we adopted a participant observation method in data collection while the main corpus of data included video-recorded observational data, the experimenting teacher's observational notes as well as the sorting and archiving of the corpus of the students' work on and off computer. As far as the students' work on the computer is concerned we used specially designed screen capture software -called Hypercam- which allowed us to record students' voices and at the same time to capture all their actions on the screen. Trying to attend to the full range of the communicational forms that students used in the meaning-making process in data analysis we followed a multimodal discourse analysis method viewed through a social semiotic lens (Jewitt, 2009). Initially data were transcribed in a multimodal way focusing on students' situated choices of resources rather than emphasizing on the system of the available resources. In an attempt to overcome the limitations presented by a sequential organisation of data and to present simultaneously multimodal phenomena, matrices with columns were used. As a unit of analysis we used the 'multimodal episode'. The multimodal data were divided in episodes that constituted easily discernable parts of children's actions and interactions with a clear focus point (Noss & Hoyles, 1996, p. 148). Thus 'multimodal' episodes do not represent some quantifiable entity but are chosen to represent clearly the kind of activity that was going on in specific time in the classroom. As episodes have been extensively used as a unit of analysis in the framework of qualitative researches, the term multimodal has been added so as to stress that the episodes used in our analysis do not rely only on oral or written language but comprise also gestures, visual images, instances of students' symbolic work on and of computer etc. The results presented here are based on the work of one group, consisted of one boy and one girl, while focusing on the way gestures were used during the construction processes.
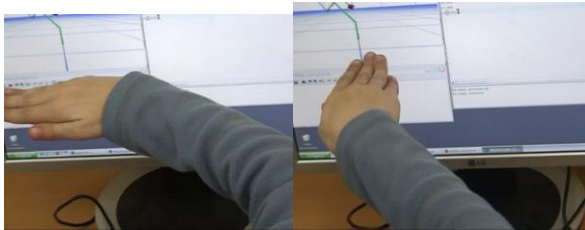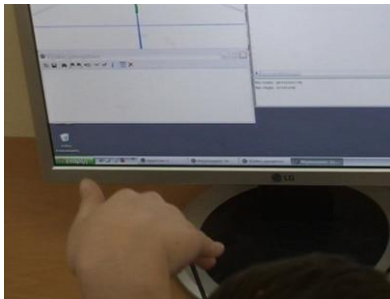
Gestures as a means of semiotic mediation

Use of gestures and turtle's navigation

It follows from the data that while the students were trying to navigate the turtle in the simulated 3d space they basically used two kinds of gestures: a) dynamic representational gestures and b) abstract deictic gestures.

It seems that the 'play the turtle' metaphor (Papert, 1980) cannot be realised physically as far as the 3d Turtle Geometry environments are concerned: In these environments the turtle moves in all the 3 dimensions without any restriction while in the real 3d space the human body can only move in a 2d horizontal plane. As a result students used extensively their palm so as to represent the 3d entity and its orientation as well as its motion in the simulated space. The palm was used as a 3d object analogous to the computational turtle, as it has distinct 'place' characteristics, up-down, forward-backward and right-left, it can be moved in all the three dimensions, while it is easily manipulated and observed. In the following episode the students are trying to decide how to carry on the turtle's journey in the 3d simulated space during the first task using a series of dynamic representational gestures. The use of the palm seems to contribute to the visualisation of a series of successive spatial representations before these representations are systematically articulated either verbally in everyday language or symbolically through logo code. Thus gestures were used as a link to the embodied turtle metaphor that underlie Turtle Geometry environments and as an intermediary stage between lived experience and institutional signs such as Logo code. Moreover these gestures seem to provide the context in terms of which students verbal
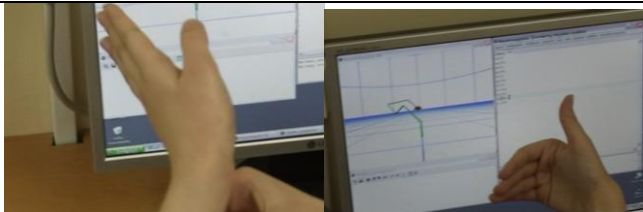
expressions are to be interpreted. It is indicative that the students' utterances cannot be understood if not accompanied by the respective gestures.

S2	Now, do you know what we should do? As it is like that, to turn it this way and to move it forward.	
S1	Not to move it down a bit?	

Episode 1: Dynamic representational gestures

In the present research the palm was not used only representationally but also deictically to indicate the turtle's direction in the 3d space. In the following episode which took place during the 1st task, the students are trying to decide how many degrees the turtle should turn so as to take the intended position and direction. As the focus point is not turn's direction but turn's measure, the palm is rather used as an indication of the various turtle's position for certain angular turns in the 3 space and in particular for the left turn of 45, of 90 and of 135 degrees.

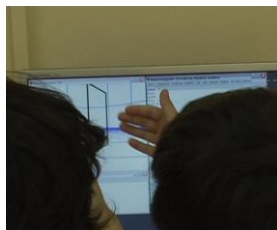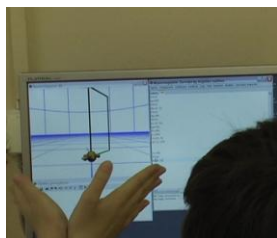| S1	Fine. We will turn it. Wait it is like that. So half of it, approximately 45, 90,	
	approximately at 135 …	

Episode 2: Abstract deictic gestures

It should be stressed that the students' palm was not used deictically as far as certain concrete objects or attributes of the context of the activity are concerned, but that these gestures integrated abstract deictic characteristics. Children's gestures rather exhibited geometrical knowledge and helped students cope with the abstractness of certain mathematical concepts such as angular turn in 3d simulated space. These abstract deictical gestures called for mathematical interpretation and implied a metaphoric use of the real space, where certain angular measures had acquired spatial properties. The aforementioned kinds of gestures - as well as the kinds of gestures that are presented in the next section- are here considered as signs that were invented and used by the students as an auxiliary means of solving the given tasks while using the specific digital tools. On the one hand these gestures are related to the accomplishment of the task and on the other hand they are rather related to the mathematical content that is to be mediated bringing in the foreground the complex relationship between digital tools, task and mathematical knowledge.

Use of gestures and 3d graphical objects' construction

It follows from the data that when the students' focus point was on the construction of 3d graphical objects, two kinds of gestures were used: a) static representational gestures and b) dynamic representational gestures.
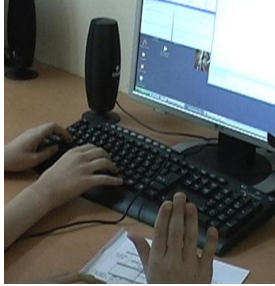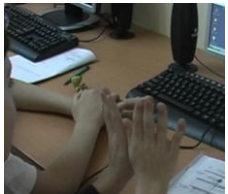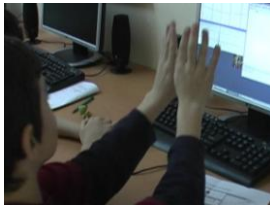
In the following episode the students are trying to translate their intuitions in visual representations so as to represent geometrical figures and their orientation. The gestures that are used could be considered as representational gestures as they have a degree of resemblance to the desired geometrical figure. In the following episode that took place during the second task, while trying to match real to corresponding virtual 3d objects, the students are using firstly the one palm to represent the position and direction of the one wall/plane that they want to construct in the 3d simulated space. Then, the intended figure and the spatial relationship between the two walls/planes are represented through the use of both palms. So the palms seem to be used as intermediary transitive objects between the real object and its figural representation on the computer screen. This kind of gestures rather depicts spatially encoded knowledge and helps students conceptualise the spatial relationships that should be then expressed in Logo code.

S1	Now as it looks this way, let's make the one wall like that.	
S2	Yes that's better.	
S1	Otherwise we can do it this way.	

Episode 3: Static representational gestures

The representational gestures used in the previous episode could be characterised as static, as they constitute static instances of the intended figures. While the children were trying to move along from static 3d representations to their design through the turtle's navigation another kind of representational gestures was noticed: dynamic representational gestures. This kind of gestures seems to represent the 3d object not as a static instance but as a result of the turtle's motion.

S2	Up (90). From this to go this way.	
S1	The staircase should rather be this way and not that way.	
S2	What? Will we do it straight?	
S1	Of course.	
S2	It should move this way. Then it should turn. … rt, no. No, there is something else, how is it called? This way and then this way.	

Episode 4: Dynamic representational gestures

In the above episode the two students are discussing about the way they should construct a staircase in the 3d simulated space of MaLT. It was a task that was carried out by the students spontaneously at the end of the second task and while they were waiting for the other groups of students to finish their constructions. Initially student 2 suggests turning the turtle up 90 degrees while representing this motion with his hand. The other student having focused not on the turtle's navigation but on the staircase's inclination in relation to the horizontal level corrects the former

showing with his palm the inclination that the staircase should have, which in any case should not be vertical to the horizontal plane. S2 reacts asking if they should do the staircase straight, understanding the other student's gesture as a straight inclined line. Then, he represents both the turtle's motion and the staircase moving his both palms. The palms side by side represent the horizontal and vertical planes of the staircase, while the hands' motion seems to represent the turtle's motion in the simulated 3d space of MaLT. In parallel he tries to translate verbally in Logo code the turtle's motion trying to find out the right turn order saying indicatively: *'Then it should turn. ... rt, no. No, there is something else, how is it called?'*, looking apparently for the 'downpitch' order which corresponds to his hands' motion. It is rather interesting that the spatial arrangement of the plane's as well as the angular turtle's turns are initially represented visually and kinaesthetically and then verbally. The use of gestures was an alternative sign, an alternative way of embodying and organizing information that the student was not able to express in purely verbal or formal ways. It should be stressed that these situated gestures denoted the intended figure not so much pictorially but through actions and as a result of it while playing a mediating role between internal, subjective imagery and shared conventional logo code.

Conclusions

In the present research gestures were understood as signs/symbols that mediated the mathematical knowledge integrated in the computational environment (Radford, 2009, Arzarello et al., 2009). Gestures were used in order to objectify, to attribute meaning to mathematical contexts and contents interpersonally and intrapersonally. A virtual gesture space was created in front of the students as a result of the use of gestures, where the various represented mathematical objects were placed, processed and interconnected. This virtual space was 'endowed' with mathematical meaning that was accessed and visualised kinaesthetically. Thus, gestures rather provided an intermediary stage between real and computational objects that fostered imagery focusing on images' structure rather than on accuracy (e.g. the exact degree of turtle's turn) that rather reduced some of the cognitive load of problem solving. In parallel, gestures offered a context without which students' verbal expressions could not be interpreted.

The gestures that students used seemed to have helped bridging the cap between abstract mathematical notions and sensorimotor experience. In the present research gestures were conceived as a way of revealing unconscious aspects of concepts formation, while certain kinds of gestures were rather strongly related to the embodied metaphors that underlie Turtle Geometry environments. Dynamic representational gestures were used not only in order to represent the turtle's motion through a series of successive spatial images but also in order to represent 3d geometrical objects as a result of the actions of a moving entity. Moreover, it follows from the research that the students used different kinds of gestures according to their point of focus. While they were focusing on turtle's navigation through body-syntonicity and the graphical results of this navigation, the students used dynamic representational gestures. On the contrary while they were observing 3d space as external observers and not through the turtle metaphor they used either abstract deictic or static representational gestures. The kind of gestures used in the various phases of geometrical objects' construction processes could rather be integrated in the broader research interest on the perceptions students have in 3d virtual environment (Hauptman, 2010) and the spatial dimensions of interactions though 3d avatars (Petrakou, 2010). Highly visual 3d Turtle Geometry microworlds, such as MaLt, seem to influence not only the kind of geometrical problems posed to students but also and most importantly the way students interact with the medium and the solution processes followed by them (Hollebrands et al., 2008; Jones et al., 2010). In this framework gestures serve for students as signs that mediate mathematical activity

and knowledge and for researchers as a window that offers a new perspective on how learners think and talk about mathematics.

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