EMBEDDING MATHEMATICS IN SOCIO-SCIENTIFIC GAMES: THE CASE OF THE MATHEMATICAL IN GRAPPLING WITH WICKED PROBLEMS

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This paper discusses the ways in which digitally enabled transformation in mathematics education could envisage a role for rationality in post-normal science and wicked problems. The scene is set firstly by reviewing the ways in which digital media have been designed and used in transformative mathematics education as a rationale for thinking about such media for wicked problem education. The problem is set in epistemological terms, can normal science approaches contribute to post-normal science. Taking into account the basic arguments regarding wicked problem education, I focus on the discussion of a specific constructionist digital tool called 'ChoiCo: Choices with Consequences', designed to embed mathematical ideas, facilitate mathematical reasoning, yet be about grappling with wicked problems. The final section discusses student discourse to set the scene for what such reasoning might look like in the context of grappling with wicked problems.

Keywords:

WICKED PROBLEMS AS A CHALLENGE FOR TRANSFORMATION IN MATHEMATICS EDUCATION

In recent times, every one of us feels exposed to wicked problems, those universal ill-defined, controversial, complex, value-laden socio-scientific issues such as the pandemic and climate change. Our society is replete with individual and shared stress, denial and inertia, ultimately leading to exponential augmentation of risk for wide-ranging consequences. In Europe, at least, there is a push for educational transformation aiming to provide students with experience in grappling with such issues in a knowledgeable contributory way. In this paper, I discuss the potential role for using digital media to engage in rationality and mathematical thinking as a means of grappling with such issues. Pedagogical transformation is not new to mathematics education, albeit in different ways. So, could mathematical rationality in handling wicked problems be one of the transformation avenues worth addressing in mathematics education?

It has now been 50 years since Papert introduced the idea of fundamentally changing students' experiences with mathematical reasoning through the use of digital media to express, explore and generate mathematical meaning (Papert, 1972). The need for transformation in mathematics education has since then been widely argued from many angles beyond the advent of digital technologies. It has been generally portrayed as a need to move away from overbearing 'visiting the works' paradigms, as Chevallard (2012) would put it, where students are typically exposed to abstract mathematical truths in a rigid, control-oriented, time-bound setting aiming to strengthen their ability to respond to specially pre-designed tasks (Riling, 2020). Instead, the push has been to find ways to provide students with agency (Andersson & Norén, 2011), with experiences in mathematical reasoning for themselves, for meaning-making in personally relevant individual and discursive settings and digital media have been perceived as powerful tools to that end (Noss & Hoyles, 1996) [1]. This powerful way in which learners use digital media to structure mathematical knowledge-in-use has been well researched and established as a goal and a challenge for transformation in mathematics education (Noss & Hoyles, 2017).

From an epistemological point of view, it is not the nature of mathematics that has been challenged but rather the ways in which the practice of mathematicians has been understood and communicated. Mathematical epistemology has not been fundamentally debated. Mathematics has been deservedly perceived as the ultimate scientific endeavor; it is the field where reasoning comes from, where rigor comes from, where the ability to make connections, to deduce and to prove, to generalize, to be certain, or to gain accuracy and develop a sophisticated language about uncertainty. This is a science where we know when something is true, and we question whether something is true in very rigorous ways (Davis & Hersh, 1981).

The transformation sought has thus to do with education, not the scientific paradigm. It is to provide learners with the opportunity to experience what it means to do mathematics, the same kind of experience mathematicians themselves go through. Mathematicians expose ideas and propositions to peer scrutiny and refutation attempts. They thus perceive mathematical ideas to be fallible, and only the ones which survive this scrutiny remain as mathematical certainty (Davis & Hersch, 1981; Lakatos, 1976). This means that most of the time they spend scrutinizing ideas by others or having their own ideas put in the frying pan so to speak. The scrutiny process is a fundamental part of doing mathematics, and transformational approaches in mathematics education argue that learners should be given much more space to engage in this kind of process.

Recently, however, and importantly highlighted by the era of the pandemic, we have realized that what has hit each of us in our society and everyday life is the engagement, pre-occupation and involvement at a personal level not with clean, potentially solvable, mathematical problems but with very complex issues and problems that are around us: climate crisis, sustainability, sustainable cities, pandemics, personal diet combining health and well-being. These kinds of issues do not really have a solution in them, and there is not any clever way in which we can find the way to deal with them, nor can we find how to cope with them and get rid of them in the end. Even though mathematicians produce endless models of such complex situations, none of them really explains the respective phenomenon in any comprehensive, resolving way. So, at the individual and social citizen level, what is required is that we become a little less stressed about these issues. How can we learn, as citizens, to grapple with them in order to survive within contexts where these issues apply. And how can mathematical reasoning and scrutiny maintain and enhance its perceived value in situations where it could play an important yet not primary role regarding the issues at hand.

In this paper, I address a question which I believe should be put to the mathematics education community:

- Is there a role for mathematics and mathematical thinking in coping with complex, contentious, socio-scientific issues?
- If yes, how can digital media be designed and used to introduce mathematical thinking and rationality in addressing and grappling with such issues?
- How can we think of pedagogical mathematical transformation with digital media to include grappling with wicked problems?

Such issues have played a central role in creating big currents pushing for change that affect educational systems in Europe and around the world. They have been connected to the ideas of cultivating 21st-century skills and action competence. Mathematics education researchers have connected those kinds of skills—such as creativity, computational thinking, collaboration and communication, problem posing and solving—to mathematical reasoning. At the same time, however, the push for educational reform has originated from much wider educational and societal perspectives and is a part of the European strategy for education and equity. The EU strategic plan for 2021–2024 is oriented towards the twin green and digital transitions for a sustainable, fair and more resilient economy and society (European Commission [EC], 2021). The development of a high-performing education ecosystem through the digital transformation of educational paradigms is also one of the core goals of the EU Education Action Plan for 2021–2027 (EC, 2020) and of UNESCO's Education 2030 Framework for action.

So, is there a role for us as a mathematics education community to contribute to this wave of change, or is it better that we stay on the side in the hope that our siloed domain of mathematics and its beauty will remain and be respected as in the previous century? In other words, is there a role for mathematical thinking and rationality in post-normal science? If yes, what kind of digital media can be used for expressing mathematical reasoning while grappling with wicked problems? How can we design for added pedagogical value based on their use?

The following three sections provide a background to carefully address these questions. First, the ways in which digital media have been designed and used in transformative mathematics education are analyzed as a rationale for thinking about such media for wicked problem education. The next two sections set the ground with respect to epistemology, how can normal science approaches contribute to post-normal science. The third then sets the scene and basic arguments regarding wicked problem education. What follows is the description of a specific constructionist digital tool called 'ChoiCo: Choices with Consequences', designed to embed mathematical ideas, facilitate mathematical reasoning, yet be about grappling with wicked problems. The final section discusses student discourse to set the scene for what such reasoning might look like in the context of grappling with wicked problems.

DIGITAL MEDIA AS TOOLS TO TRANSFORM MATHEMATICS EDUCATION

Let us, in this section, look a little more deeply at the ways in which digital media have been perceived and designed to bring added pedagogical value in transforming mathematics education paradigms to cultivate mathematical reasoning (Bray & Tangney, 2017). Researchers seem to agree that there is particular value in digital media being used by students as tools with which to engage in mathematical reasoning, in putting mathematical concepts and ideas to use, in mathematical discourse and expressivity. When we have classrooms where students are given space to develop their own ideas and to work with these tools, we can see that the mathematical meanings that they develop and construct are unavoidably connected to the tools that they use. Researchers have witnessed this kind of reciprocal shaping of meanings and tools when mathematics is put to use to create and change mathematical models and representations (Noss & Hoyles, 2017; Artigue, 2012). The pedagogical value in meaning-making has been considered as important enough so as to address the connection between those meanings and the abstract curricular mathematical concepts as a necessary educational task in the context of students having built a positive disposition towards and experience with mathematical reasoning.

As learners create models and representations with these tools, they progressively create 'schemes of action' as Vergnaud (2009) would put it, i.e. individual and shared meanings of a tool's functionality and kinds of use together with the kinds of mathematics cultivated during such use. Thus the key aspects of focus in designing such tools and envisaging their usage are:

- Mathematical expression, augmenting the representational repertoire and interdependencies

- Engagement with mathematical thinking
- Putting concepts to use
- Reciprocal shaping, instrumentalization, constructionism and creativity

The task is to generate environments that are rich in opportunities for meaning-making, to perceive digital artifacts as media for expressing mathematical meaning, to access powerful mathematical ideas that are otherwise difficult or obscure with pencil and paper or with other representations and to engage teachers in taking part in the design of pedagogically added-value activities. The main concern in looking for pedagogical added value is thus connected to designing for innovation to adopt a transformative stance to education. The main thrust in this approach questions the way that mathematics is perceived and taught and the way that curricula are structured and looks for ways in which we can use technology within a transformation process. As researchers, we focus on technology for expression and meaning-making and we are developing theory on meaning-making processes and on teachers' knowledge and teachers' practices. Our main concerns have thus been to:

- generate environments rich in opportunities for mathematical meaning-making (Papert, 1972);
- perceive digital artefacts as expressive media for mathematical meaning-making, a new literacy (Noss & Hoyles, 1996);
- access powerful mathematical ideas otherwise obscured by traditional methods of expression (diSessa, 2000; Willensky & Papert, 2010);
- engage teachers in designing added-value media and activities and dealing with professional, institutional and societal traditions so as to generate such environments in the classroom (Ruthven, 2014); and
- develop media that is specially designed for questioning traditional practices and doing something different.

At the Educational Technology Lab we have been adopting a transformational approach by designing and using media for teachers and students to in turn design and tinker with models and representations. Over a period of more than 25 years, we have been engaged in design research to illuminate mathematical processes in respective educational practices and to contribute to the development of a 'framework for action' theory helping to both design and understand meaning-making (diSessa, & Cobb, 2004). In this venture, we found it most useful to combine and integrate diverse theoretical constructs, having been greatly influenced by our participation in the TELMA, ReMATH and M C Squared European Research projects whose main aim was precisely to forge connectivities amongst constructs lying in fragmentation on a 'theoretical landscape', to use Artigue's terms (Artigue & Mariotti, 2014). Our particular objective was not the practice of creating such connections per se but instead of considering how to best try to make sense of the environments we designed and studied. So, we found these particular four constructs, albeit widely diverse, to be pivotal in our approach.

1. Conceptual fields (Vergnaud, 2009)
2. Restructurations (Willensky & Papert, 2010)

3. Half-baked artefacts (Kynigos, 2008)
4. Reciprocal shaping of meaning and tool (Hoyles et al., 2004)

We found Vergnaud’s idea of conceptual fields centrally useful in the sense that, for mathematics education, it diverts priority from a mathematical concept to be ‘learned’, to all that makes it useable and communicable. To think, i.e., of a concept in educational design, it is necessary to place it in the center of a dense circle of related concepts and a set of representations that become the basis for resolving a set of problem situations. So, in education, this is the way we should be thinking. We should not be thinking of whether students learn how to factorize or learn how to solve a quadratic equation, but rather of situations resolvable by dense sets of concepts around a central one.

Restructurations is the exercise of questioning the structure of the curriculum and the kinds of mathematics to best approach mathematical problems. The current curriculum structures have been decided, established and fixed in historical time before the advent of digital media and even before the advent of mathematics education research, for that matter. But mathematics is the discipline characterized by fluidity in the ways in which it makes sense to build structures; its nature is such that you can portray mathematical concepts in a very large number of different alternative structures. So now that we have technologies and we live in the technology world, it is time to rethink about what kind of structure of mathematical concepts is now amenable for children to engage in mathematical thinking with these tools. Imagine, for instance, a section on ‘curvature’, on periodicity, on rate of change, inflation, compound interest and approximation combined, on mathematical complexity, on gaming theory. Ask the question: which mathematical structures are good spaces for students to engage in meaning-making and mathematical reasoning, given digital media?

Half-baked artifacts; well, this is didactical design, or rather, engineering. It is when, from a pedagogical point of view, students are given problems, models or representations that are incomplete or have faults in them and then invited to identify and correct them. Behind this, there is the epistemological idea of fallibility, the idea of questioning and the idea of not perceiving mathematics as a game of absolute truths but perceiving mathematics as a field where reasoning and questioning prevails.

So, the main concerns of the research community at large, and of our Educational Technology Lab, have seen mathematics as useable intellectual processes and traits in diverse situations. Educational transformation in mathematics education has perceived digital media as a pivotal tool and digital transformation in society as the global situation in which this educational transformation may start to materialize. In this wake, mathematics curricula and curricular structures have come into scrutiny, asking the question: what kinds of structures can operate as fertile fields within which learners can develop mathematical reasoning. With respect to digital media, mathematical reasoning has been connected to constructionism, a kind of discursive low-stakes tinkering-style engineering co-evolving with computational thinking skills and competences. However, still in all cases, the mathematics education research community has understandably perceived mathematics as the priority and the end target. The situations, the tools and the restructurations have been a means to an end.

NORMAL VS POST-NORMAL SCIENCE

The problem is with normal science. Scientists are perceived by the wider society with diminishing credibility and relevance, as people who will give you facts and truths that are not so relevant when you think of the issues at hand. So what happens? So how else can we think of science? Well, recently, there has been a movement termed 'post-normal science'. It addresses complex issues of our time where disciplinary fragmentation and traditional scientific conduct appear to lack the necessary capacity to allow an integrated understanding of the issues. Transmitting simple truths does not help policy makers, and that does not help citizens and the individuals. We need to draw on epistemological principles that recognize uncertainty and develop ways of dealing within uncertainty, within value-laden agency.

Most current science thinking and research, centrally including mathematics, has been built on epistemological assumptions developed along the deliberate aim to moderate complexity and minimize uncertainties in the world so that problems and issues can be ultimately modeled. These 'normal science' paradigms (Kuhn, 1962), supported by appropriate conceptual modes of representing reality and specialized codes for studying it, narrow down the focus of their enquiry within the boundaries of specialized disciplinary fields such as mathematics to address attentively defined (but eventually simplified) 'problems', with the intent to generate valid and generalizable evidence-based knowledge to feed decision-making. However, when it comes to complex issues of our time, disciplinary fragmentation and traditional scientific conduct as encouraged by 'normal science' seem to be lacking the necessary capacity to allow an integrated understanding of the issues. Moreover, transmitting simple truths to policy making is rather inadequate when dealing with multi-faceted issues carrying a great degree of uncertainty (Heazle, 2012). There are many open questions as to how science can contribute to fostering social innovation and change in as many social groups as possible, rather than providing only expert-based knowledge to policy makers. This is particularly the case with, for example, current crisis and sustainability challenges, recognized as complex, controversial, and value-laden issues by nature and, therefore, difficult to be dealt with in mono-disciplinary ways. Complexity stems from their multi-faceted character and the requirement to apply various perspectives to grasp them more holistically. Different interpretations may lead to different implementations based on the context and the situation. These features render such wicked problems difficult for normal scientific practice to address and deal with. To counteract these shortcomings, 'post-normal science' has emerged as an alternative paradigm of scientific enquiry and knowledge (Funtowicz & Ravetz, 1993). Drawing on epistemological principles that recognize uncertainty, value-laden agency, and context-specificity as intrinsic attributes of the contemporary, post-normal science promotes transdisciplinary approaches to framing and studying current complex issues and gaining an understanding of the world. Global crises, pandemics and sustainability issues are among those most characteristic examples the understanding of what necessitates the application of post-normal lenses and processes, such as the co-creation of diverse types of knowledge, the employment of participatory methods, designs, and tools, that facilitate the emergence of multiple representations and reflection to take place.

WICKED PROBLEMS AND WICKED PROBLEM EDUCATION

Consider the role of schooling to inspire lifelong citizen engagement with 'wicked' problems that can contribute to a democratic, socially engaging sustainable development practice, where experts and various groups of citizens with different perspectives engage in a dialogical inquiry on a complex, fuzzy, multi-faceted, contentious issue, such as sustainable living. This kind of issue has been called 'a wicked problem', i.e., a dysfunctionality within a complex system (Conklin, 2006).

Wicked problems are difficult to contain and structure, are interconnected and interdependent, are ill-defined and dynamic as their parameters are continually in flux (Rittel & Webber, 1973; Coyne, 2005). Individuals often feel overwhelmed, develop denial and resignation to such a problem, followed by inertia due to a sense of determinism, which permeates societies (Lazarus, 2009; Hulme, 2009). Yet wicked problems need action at many levels, present inertia risks and exponential growth of the problem and its consequences at high stakes (Brown et al., 2010). For the individual, it is important to engage in becoming sensitive and knowledgeable on the problem and to also engage in actions such as taking care of individual footprint, challenging own actions, beliefs and habits, being interested not only in individual action but also in contributing to collective action at a level of the city or municipality (Cantor et al., 2015). A paradigm shift is needed, from solving well-defined siloed problems to a post-normal science approach (Lehtonen et al., 2019). So, consider a transformational stance to schooling in an attempt to integrate such a post-normal science approach in teaching and learning, addressing and perceiving students as young citizens (McLaren, 2013). Consider the challenge of harnessing wicked problem education to become syntonic and integrated with the innovative educational push towards cultivating the eight key competences for lifelong learning.

- Agency, ability to make own decision, challenge not set by another (Kynigos & Diamantidis, 2021)
- Action-in-context, when the context is not necessarily about mathematics in school
- Beyond silo disciplinary approaches
- Beyond timed, solvable, regulated challenges
- Beyond integration: a competency (Geraniou & Jankvist, 2019) in the service of another

So what is a wicked problem exactly? It is a problem impossible to solve, and that is because it is not well defined. It is contradictory. Different people have different views on it. It changes all the time. It connects to different things. Conklin (2006) called it a dysfunctionality within a complex system. Such problems are difficult to contain and structure; they are interconnected, interdependent and ill-defined. And their parameters are continually in flux. Some examples of wicked problems are poverty; urban renewal; school curriculum design; education, environmental and natural resources policy; healthcare; climate change challenges; sustainable cities; diet; individual and social challenges in times of world crisis.

So, these wicked problems cause problems to the individual. They often develop a denial about the problem. Assertions such as ‘come on, pandemic corona-virus! It’s easy, it’s just a flu!’. They create resignation of the individual regarding the acknowledgment of the existence of the problem. We see such a point argued all over the media, ‘there’s no point in vaccinating since if you’re vaccinated you can still catch it’. Resignation is followed by inertia; ‘I’ll wait, I’ll wait for everybody else to get vaccinated and then see what happens’. But wicked problems need action at many levels because the inertia is a risk, and there is an exponential growth of the problem if people do not realize and do not start developing strategies. And the consequences are high stakes. So, for the individual, it is important to become sensitive and knowledgeable not to find a solution and also to engage in actions such as taking care of the individual footprint, but also perceiving that they are a member of societies at different levels who are collectively addressing the problem.

To date, transformation in mathematics education has hardly addressed the role of mathematics and mathematical thinking in post-normal science. What can a normal science such as mathematics provide to empower and support post-normal perspectives aiming to address controversial ill-defined, complex socio-scientific and value-laden issues? Fallibility in mathematics has been perceived in the context of the process to look for truth, for certainty, in a normal science setting. In post-normal science, however, the focus is not on the process of producing mathematical truth but could potentially be on the role such truth can play in, say, wicked problems. In this paper, I suggest that mathematical thinking and mathematical concepts do and should have a secondary but no less important role to play in post-normal science and that mathematics educators need to consider their involvement in the recent pedagogical wave demanding schooling to afford wicked problem education.

DIGITAL MEDIA TO GRAPPLE WITH WICKED PROBLEMS: THE CASE OF CHOICO

The first section in this paper contains a discussion of how digital media have been designed as tools to help with the pedagogical wave of transforming the mathematics education paradigm from what Chevallard (2012) calls a 'visiting the works' paradigm to an experiential, questioning the world, creative and discursive paradigm. The transformations, however, have so far maintained mathematics as the primary educational objective of the enterprise where the focus is on the modeling of mathematical objects and representations outright or at most the modeling of objects and behaviors directly and importantly embedding mathematical concepts (Artigue, 2002; Kynigos, 2018; Sarama & Clements, 2002; Kaput et al., 2002; Sinclair & Freitas, 2014). Even in the case of media which is primarily focused on computational thinking and creativity to create games such as scratch, mathematics educators have shaped microworlds and modeling exercises with a focus on the mathematical concepts inherent within (Benton et al., 2016; Cader, 2018). The most well-known attempt to design a medium for students to engage with complex issues is NETLOGO (Willensky, 2020). Even there, however, the focus is on mathematics as a means to fully understand the phenomenon by modeling it, based on an albeit diverse kind of mathematics calling for a restructuration of our perception of mathematical curricula (Willensky & Papert, 2010). In this paper, the attempt is to consider digital media primarily in the role of tools to help grapple with wicked problems, yet, at the same time, embedding mathematical ideas and designed to cultivate rationality in such an enterprise.

It is in this context that we introduce a constructionist tool which we call 'ChoiCo', a digital medium specially designed for post-normal science education. ChoiCo is an acronym for 'Choices with Consequences' (Kynigos & Grizioti, 2020). It is a system for authoring games embedding socio-scientific issues. The system leaves the choice and definition of such an issue up to the user. It is based on the gaming idea that there is a single gamer making choices amongst objects placed on a geo-coded map. Every choice has consequences across a pre-set range of fields, yet there is no clean choice, i.e. one which has only positive or negative consequences. The game ends when the player crosses some pre-set value in one of the fields, i.e. crosses a 'red line'. So the gamer needs to navigate through a field of choices, the point of the game being to stay on the game as long as they can, avoiding 'red liness'. The more choices made, the better the player. Sustainability is key; the more the player can sustain making choices, the better. But most importantly, ChoiCo affords important transparency, leaving users, in the role of game creators or modifiers, to name as many fields as they wish, to set values for every choice, to program the starting values, the 'red lines' and a number of 'warning messages' and other rules via a block-based programming language. Field values can be numerical fixed or random, visible or hidden from the player (they can make the player need to infer the consequence of the field by observing some text, a video or a picture).

ChoiCo is thus not just about designing or playing a game, it affords the user to take on the role of a prosumer, someone who engages in-game modding as well as design and play, interchangeably.

Each game can be considered as a ‘living document’ to use the term coined by Trouche and his colleagues in their theory of addressing educational practice through the continual re-design of educational resources (Guin & Trouche, 1998). The main features of ChoiCo are based on the following design principles.

- Constructionist games, games affording access to the content and rules of the game and providing tools to define and to change them
- Free climate – alternative reality, low stakes, so that users find a safe space to try out risk-free solutions to wicked problems and consider the consequences
- A framework – reference for discussion and debate, the idea is that pairs or groups of students engage in the modding process
- Rules and content of a game open to modding
- Gaming rules: sustainability, i.e. stay on the game as long as you can
- Interchangeable gameplay and game modding, i.e. the practice of adopting a binary role of player and designer of a game

So, let us start with an example of a game that was actually designed by postdoc researchers at the Lab (Grizioti et al., 2021). Consider a user in the role of the player; let us give her a name, ‘Mary’. Mary is a citizen, and she lives in the covid pandemic era. She has some choices to make on what to do in her day. For instance, she can consider running. If she does choose to run, the game tells her what the consequences will be along a line of values. The values are ‘physical capacity’, ‘it’s fun’, ‘social’, ‘money’ and ‘risk of covid infection’. So the game tells Mary what is going to happen if she makes a choice before she decides to make it. If she chooses to engage in running, then the covid risk would be a random number from minus 15 to minus 20, i.e. an equal probability in the respective range of values. So it would not be much of a risk, but still, Mary would not be certain of what is going to happen. The ‘physical condition’ consequence is a function of how much physical condition Mary would have if she was walking. And the others are just numbers. So, upon clicking on the selection of the choice to go running, Mary observes the change of aggregate values in a respective panel on the screen. Then, she can try doing something else. For example, she can try going to the local store (Figure 1).

Figure 1. The consequences of going to the local store

That choice would result in a much larger covid infection risk, 15 to 25, and would result in spending money (-30). On the positive side, it would up the social and physical condition, a lot of walking up and down the isles you see. If she then chooses to go to the mall, she would be in danger of being thrown out of the game since the covid risk there would be immense. So maybe she might decide going to her home instead. Clicking on that choice brings up a different plane with things to do at home, e.g. work from home, shop online etc. For instance, if she decides to sleep and continues to select that choice effectively to sleep through the pandemic, the game quickly sends a warning sign 'You're unhappy. You need to do something.' and then if she continues, throws her out since the 'fun' value crosses the line and the game reports that she has become depressed. So the idea of the game is to stay on the game as long as possible, and there is no choice that has only positive or only negative consequences. Mary needs to find a way to navigate in order to stay in the game as long as possible. This is a game about a citizen dealing with a wicked problem.

And now about modding a game, taking on the role of a hacker. In the example of the covid game, everything mentioned so far can be changed by the player changing roles and becoming a game modder. A button click switches to the editing page where Mary herself or anyone else can make changes at all levels. Change the picture. Add new choices. A new choice appears on the map and also on a tabular representation of a relational database as a new record. Mary can give it a name and start putting values on the consequence fields. She can observe that there are diverse kinds of values she can allow for each of the field columns. For instance, the covid risk field has been defined as containing random values in a range set by Mary. There are other formulas she can use to define field values, simple functions or just straight numerical values. Mary can also easily change or add fields (Figure 2). Finally, she can switch to a Blockly programming feature in order to write programs to set the initial values, set the 'red lines', put up warning texts or sounds when a certain value gets close to a red line and anything else regarding the game rules. (Figure 3).

Figure 2. Making changes to map, choices and their consequences

All of these affordances have been designed to allow users to engage with a larger and more complex set of concepts, practices and values than one may find in like-minded authoring systems in education. There are three distinct but also interconnected areas for these. One is, of course, the socio-systemic issue embedded in a game; the other is the computational thinking, i.e. concepts and practices cultivated and employed, and the third one may involve mathematical concepts and rationale. Before we elaborate on the latter, it is important to say a few words about computational thinking since this too can be connected to mathematical thinking in various ways (Barr et al., 2011). ChoiCo affords the use of functionalities to do with geo-coded data with relational databases and with block-based programming giving some emphasis on event handling and boolean logic.

This enables the main characteristics of computational thinking as originally defined by Wing (2006).

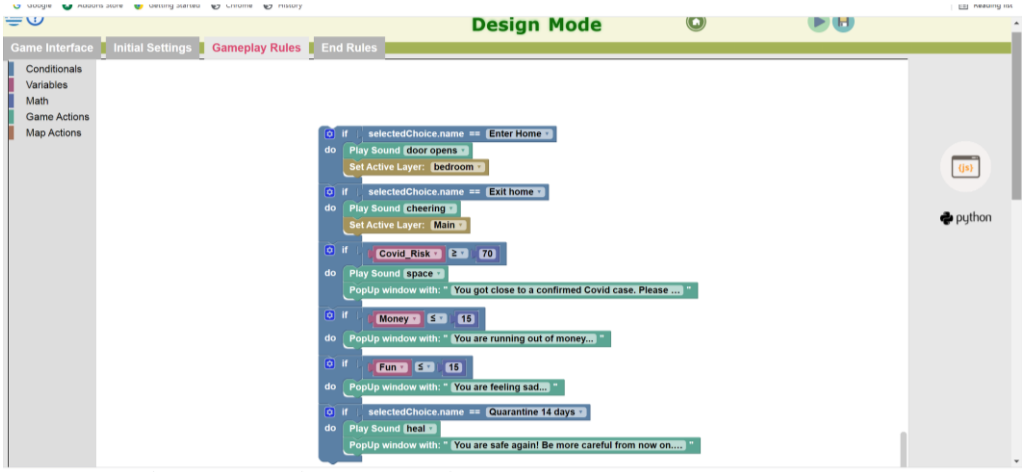

Figure 3. Changing the rules, initial values, warnings and red lines

In a study of students using ChoiCo in such a way (Kynigos & Grizioti, 2020), we analyzed their progressive process of exploration (play the game), deconstruction (break down the game structure), analysis (analyze the game elements), synthesis and construction. The process involved the integration of interacting with various affordances (graphics, story, rules, characters, etc.) and with the use and understanding of Blockly programming concepts such as conditionals, boolean logic, event handling, but also programming processes such as developing high thinking skills like iteration and refinement, debugging, error prediction, etc. (El-Nasr & Brian, 2006; Moshirinia, 2007; Salen 2007).

THE MATHEMATICAL IN GRAPPLING WITH WICKED PROBLEMS

So how can we think of the use of ChoiCo to develop dispositions to use rationality and mathematical thinking in order to grapple with wicked problems such as the consequences of the covid pandemic? Let us first consider the Covid game—and ChoiCo more widely—as a mathematical microworld (Healy & Kynigos, 2010). What mathematical concepts are or could be embedded in a game? And when students change the game, what mathematical reasoning could they engage in as they identify, question and modify values and relations between them? As implicitly discussed in the previous section, the Covid game embeds proportional thinking, functions, probability, mathematical issues related to programming. But, of course, these mathematical ideas adopt the status of affordances. When the game is put to use by learners, the identification of these ideas and the ways they may or may not be put to use is a process of instrumentation and instrumentalization (Artigue, 2012).

Game modding in education has mainly been connected to computational thinking, but if the games have embedded mathematical ideas, then this computational thinking becomes connected to thinking mathematically. ChoiCo games are thus seen as productions; they are artifacts designed to be used by somebody else. They are fun and have many different kinds of connections: connections to real issue debates, connections to gaming, connections to entrepreneurship. They could therefore be considered as useful resources in this new era of 21st-century skills and equity and wanting to change this silo domain structure of the education system. And they still retain some of the benefits that we have from teaching students domains and mathematics in specific.

ChoiCo games can be resources for learners to engage and to grapple with socio-scientific issues in the context of post-normal science and they can be specifically designed to invite students to make changes. They need not be designed for changes to be difficult and for students to be terribly savvy. They can be designed for students to find it easy to engage in modding. From a pedagogical point of view, rather than resources designed to enhance student responsiveness to pure mathematical tasks, they can be considered as tools to cultivate mathematical disposition and competence, perceiving students as young citizens. Modding with games such as ChoiCo can then help students to recognize the value of mathematization, modelling, tinkering with models, using rationality to grapple with wicked problems

There are a number of freely available games already up there on the ChoiCo site, http://etl.ppp.uoa.gr/choico. Furthermore, a hitherto small number of ChoiCo games designed for more focus on mathematical ideas around shopping in a supermarket reside in a space with large-scale visibility and use in the Greek Education system (http://photodendro.edu.gr). This is an infrastructure based at the Ministry of Education called ‘the digital school’, which contains a large portal of digital artefacts for students to use in all subjects and at the same time has links to these artefacts residing inside the online version of the curriculum books from year 3 to 11. There are around 1600 such artefacts in mathematics, 1200 built with GeoGebra, 220 built with an ETL grown 3D dynamic programmable modeller called MaLT2 (http://etl.ppp.uoa.gr/malt2). There are six versions of the Supermarket ChoiCo game spread at the end of primary and beginning of secondary year books.

The links to ChoiCo games in the curriculum book is in a section about mathematical problems. Clicking onto the ChoiCo micro-experiment (Kynigos, 2020) gets you directly into the game, which is about doing things in a supermarket. Your values are a number of items, how much money you have, health and pleasure (Figure 4).

So if you buy chocolate then you get: price is 7E, number of items is 7 times more than honey, health is -5, pleasure is 4. So if you keep eating only chocolate, the game will warn you when your health is below 10 and throw you out when it is below zero on the grounds of poor health. If, instead, you buy yogurt or broccoli, your health gets better, but your pleasure is reduced. The idea here is that consequences do not only have direct values. Sometimes values need to be considered in connection to other things, so students need to think about units of measure and proportional relations and operations. The Supermarket game is an example of shaping the design of a game to fit the mathematics more directly within a silo mathematics curriculum. But still, even now, there are important value-laden issues such as taste, health, pleasure eating, balanced spending etc.

Figure 4. The Supermarket Game

The idea behind ChoiCo is what we at the Lab call ‘black and white box design’ (Kynigos, 2004). So there are some digital objects and functionalities that are black boxes to the user, such as the database. ChoiCo has not been designed to get users to reprogram a relational database. It is unlikely for a piece of technology to contain programming and databases and a GIS at the same time. The design principle, however, is to prioritize a pedagogical perspective irrespective of whether it's easy to find the technological infrastructure. This is dealt with by finding available components keeping the development part mainly in gluing them together, and then building whatever else is needed on top (Kynigos, 2004).

WHEN MATHEMATICAL REASONING AND VALUES CO-EXIST

So what does mathematical reasoning in the context of grappling with wicked problems sound like? In the example elaborated here, a group of three 13-year-olds are jointly modding a ChoiCo game designed by the researchers to include questionable perceptions about what it means for a citizen to live a life-supporting sustainability in their city. The students played the game to start with and then began to discuss the ideas embedded within. R is the researcher, S(x) is a student.

R: So who is the winner?

S8: We all are! We all finished the game.

S1: We won because we have the largest amount of money: 1200.

S8: <u>Who said that money was most important for the game?</u> All you did was to go from work to home and vice versa.

S5: I think we won ‘cause we have the highest energy levels and the highest social status.

S11: Wait, wait. The winner is the one who has the highest values in all these: money, energy, fun, social status, health, hygiene.

R: What kind of life do you have to live in order to achieve that?

S16: <u>You have to do a lot of everything: have a lot of fun, have a lot of money, do not neglect your social life…</u> This is all too much.

S3: You have to be a freak to live like this; you won’t have a moment of peace.

In the above dialogue generated by the researcher, the students discussed over a gaming idea—what does it mean to win—connecting it primarily to the wicked problem at hand, is a citizen's life worth living if they go for high achievement in all aspects all the time. Within this cycle, the students considered numerical values and their aggregates, implicitly keeping in mind that each of the available choices had at least one undesirable value, a negative number in this respect. The underlying problem was how to increase the values of all the fields even though every choice would unavoidably bring a decrease in at least one of them.

S1) I am not sure about not having cars in the city.

S(2) I am telling you it has been done in Freiburg. Cars are related to pollution.

S(1) Yes, but imagine how much more time you need if you go to work by bicycle. You need to wake up at least one hour earlier.

S(3) Ok then, we will add time in the indicators. Taking the bicycle should have reduced pollution but raised time.

In discussing the pros and cons of ways to go to work, the students identified that pollution and time were conversely dependent and thus decided to insert another field, time. Here again, their focus was on the pollution problem primarily, and the inverse proportion idea was a tool with which to think about and argue for the citizen's choice to take the car or the bicycle to work.

DISCUSSION

Admittedly this paper has opened up many issues simultaneously; can mathematics as a normal science be considered as a tool, skill and competence in situations better understood through post-normal science, such as individuals and collectives against up against wicked problems? How can the design of digital media for wicked problem education incorporate agendas from transformative mathematics education, such as those mentioned in the first section? What kinds of skills, competences and dispositions can be cultivated with the help of mathematical reasoning? Can mathematical reasoning be cultivated in the context of making one's own decisions, being creative and adopting an individual and social active stance to wicked problems?

The ChoiCo ‘citizen in the Covid-19 Era’ game example and the short excerpts from students modding a ‘citizen in a sustainable city’ game were used as contexts with which to bring some elaboration of potential educational and research endeavors. The mathematical ideas embedded in the two example games ranged from simple operations to proportions and linear functional relationships to probability. The range of ideas that may be useful in wicked problem education has yet to be understood, but most of all, the interesting question is what kind of rationality may grow using mathematical reasoning and, in particular, how can this rationality be used in the quest to understand and grapple with wicked problems. In the excerpts, we saw students reasoning at different levels, from articulating local arguments of aggregating or calculating values to re-considering the ‘big issues’ such as 'what is the value of being a high scorer in everything’. There is much more to be learned about how reasoning can be used in such situations and how and when mathematical ideas and concepts might become useful. The Covid ChoiCo game could embed a larger range of mathematical concepts such as quadratic or exponential or periodic functional relations, and a pendulant course could be designed with respect to attention and focus from citizen habits to the mathematics underlying the consequences. In any case, the paper aimed to call upon the mathematics education community to consider the role of mathematical reasoning for wicked problems and the challenge to develop an argument for cultivating such reasoning in this kind of transformative educational context.

NOTES

1. It is, of course, the case that diverse approaches and tools have been developed and tried out in mathematics education, some of them designed to maintain and enhance drill and practice effectiveness in a traditional exposition to abstract mathematics curricula.

PLENARY ADDRESS VIDEO

https://www.youtube.com/watch?v=kRr0gTcOt_s&ab_channel=FacultyofArts%2CAarhusUniversitet

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2013–2016: “Mathematical Creativity Squared' - A Computational Environment to Stimulate and Enhance Creative Designs for Mathematical Creativity”. Χρηματοδοτούσα αρχή: Ευρωπαϊκή

Ένωση, P7-ICT-2013.8.1, Technological Development and Demonstration, Strategic Objective "Technologies and scientific foundations in the field of creativity" (Project No.: 610467),

2005-2009: 'ReMath' - Representing Mathematics with Digital Media FP6, IST-4, STREP 026751 (2005 – 2008). Principal Investigator (for CTI). Project funding 2,2 M E, own institution 400 k E,

'Kaleidoscope' - Concepts and Methods for Exploring the Future of Learning with Digital Technologies, # 507838, 'TEL' - 'Technology-enhanced Learning and Access to Cultural Heritage', Network of Excellence, FP6-2002-IST Action line.3.1.12 (2004-2007). European Research Team 'Technology Enhanced Learning of Mathematics' http://telma.noe-kaleidoscope.org,

REFERENCES

Andersson, A., & Norén, E. (2011). Agency in mathematics education. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (pp. 1389–1398). University of Rzeszów; ERME.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, *7*(3), 245–274. https://doi.org/10.1023/A:1022103903080

Artigue, M., & Mariotti, M. A. (2014). Networking theoretical frames: The ReMath enterprise. *Educational Studies in Mathematics*, *85*(3), 329–355. http://www.jstor.org/stable/43589825

Barr, D., Harrison, J., & Conery, L. (2011). Computational thinking: A digital age skill for everyone. *Learning & Leading with Technology*, *38*(6), 20–23.

Benton, L., Hoyles, C., Kalas, I., & Noss, R. (2016). Building mathematical knowledge with programming: Insights from the ScratchMaths project. In *Constructionism in Action 2016: Conference Proceedings*. Suksapattana Foundation.

Bray, A., & Tangney, B. (2017). Technology usage in mathematics education research—A systematic review of recent trends. *Computers & Education*, *114*, 255–273. https://doi.org/10.1016/j.compedu.2017.07.004

Brown, V. A., Harris, J. A., & Russell, J. Y. (Eds.). (2010). *Tackling wicked problems through the transdisciplinary imagination*. Earthscan.

Calder, N. S. (2018). Using Scratch to facilitate mathematical thinking. *Waikato Journal of Education*, *23*(2), 43–58. https://doi.org/10.15663/wje.v23i2.654

Cantor, A., DeLauer, V., Martin, D., & Rogan, J. (2015). Training interdisciplinary "wicked problem" solvers: Applying lessons from HERO in community-based research experiences for undergraduates. *Journal of Geography in Higher Education*, *39*(3), 407-419. https://doi.org/10.1080/03098265.2015.1048508

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: Perspectives apportées par une approche anthropologique [Fundamental concepts in didactics: Perspectives provided by an anthropological approach]. *Recherche en Didactique des Mathématiques*, *12*(1), 73–112.

Conklin, J. (2006). Wicked problems and social complexity. In J. Conklin, *Dialogue Mapping: Building Shared Understanding of Wicked Problems* (pp. 1–25). Wiley.

Coyne, R. (2005). Wicked problems revisited. *Design Studies*, *26*(1), 5–17. https://doi.org/10.1016/j.destud.2004.06.005

Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. Birkhauser.

diSessa, A. (2000). *Changing minds, computers, learning and literacy*. MIT Press.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *The Journal of the Learning Sciences,* *13*(1), 77–103. https://doi.org/10.1207/s15327809jls1301_4

El-Nasr, M. S., & Smith, B. K. (2006). Learning through game modding. *Computers in Entertainment*, *4*(1), 7–es. https://doi.org/10.1145/1111293.1111301

European Commission. (2020). *Digital education action plan (2021–2027): Resetting education and training for the digital age*. https://education.ec.europa.eu/focus-topics/digital/education-action-plan

European Commission (2021). *Horizon Europe: Strategic plan 2021–2024*. Publications Office. https://data.europa.eu/doi/10.2777/083753

Funtowicz, S. O., & Ravetz, J. R. (1993). Science for the post-normal age. *Futures*, *25*(7), 739–755. https://doi.org/10.1016/0016-3287(93)90022-L

Geraniou, E., & Jankvist, U. T. (2019). Towards a definition of "mathematical digital competency". *Educational Studies in Mathematics*, *102*(29). https://doi.org/10.1007/s10649-019-09893-8

Grizioti, M., Oliveira, W., & Garneli, V. (2021). Covid-19 survivor: Design and evaluation of a game to improve students' experience during social isolation. In F. de Rosa, I. Marfisi Schottman, J. Baalsrud Hauge, F. Bellotti, P. Dondio, & M. Romero (Eds.), *Games and Learning Alliance. GALA 2021. Lecture Notes in Computer Science, vol 13134* (pp. 283–288). Springer. https://doi.org/10.1007/978-3-030-92182-8_30

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematics Learning*, *3*(3), 195–227. https://doi.org/10.1023/A:1009892720043

Healy, L., & Kynigos, C. (2010). Charting the microworld territory over time: design and construction in mathematics education. *ZDM Mathematics Education*, *42*(1), 63–76. https://doi.org/10.1007/s11858-009-0193-5

Heazle, M. (2012). *Uncertainty in policy making: Values and evidence in complex decisions*. Routledge.

Hoyles, C., Noss, R., & Kent. P., (2004). On the integration of digital technologies into mathematics classrooms. *International Journal of Computers for Mathematical Learning*, *9*(3), 309–326. https://doi.org/10.1007/s10758-004-3469-4

Hulme, M. (2009). *Why we disagree about climate change: Understanding controversy, inaction and opportunity*. Cambridge University Press.

Kaput, J., Noss, R., & Hoyles, C. (2002). Developing new notations for a learnable mathematics in the computational era. In L. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 63–88). Routledge.

Kuhn, T. (1962). *The structure of scientific revolutions* (1st ed.). The University of Chicago Press.

Kynigos, C. (2015) Constructionism: Theory of learning or theory of design? In S. J. Cho (Ed.), *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 417–438). Springer International. https://doi.org/10.1007/978-3-319-17187-6

Kynigos, C. (2016). Constructionist mathematics with institutionalized infrastructures: The case of Dimitris and his students. In E. Faggiano, F. Ferrara, & A. Montone (Eds.), *Innovation and Technology Enhancing Mathematics Education: Perspectives in the Digital Era* (197–214). Springer International.

Kynigos, C., & Grizioti, M. (2018). Programming approaches to computational thinking: Integrating Turtle geometry, dynamic manipulation and 3D space. *Informatics in Education*, *17*(2), 321–340.

Kynigos, C., & Yiannoutsou, N. (2018). Children challenging the design of half-baked games: Expressing values through the process of game modding. *International Journal of Child-Computer Interaction*, *17*, 16–27. https://doi.org/10.1007/978-3-319-42975-5_1

Kynigos, C. (2020). Half-baked constructionism: The challenge of infusing constructionism in education in Greece. In N. Holbert, M. Berland, & Y. Kafai (Eds.), *Designing Constructionist Futures: The Art, Theory, and Practice of Learning Designs* (pp. 61–72). MIT Press.

Kynigos, C., & Grizioti, M. (2020). Modifying games with ChoiCo: Integrated affordances and engineered bugs for computational thinking. *British Journal of Educational Technology*, *51*(6), 2252–2267. https://doi.org/10.1111/bjet.12898

Kynigos, C., & Diamantidis, D. (2021). Creativity in engineering mathematical models through programming. *ZDM Mathematics Education*, *42*, 63–76. https://doi.org/10.1007/s11858-021-01314-6

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge University Press.

Lazarus, R. J. (2009). Super wicked problems and climate change: Restraining the present to liberate the future. *Cornell Law Review*, *94*(5), 1153–1233.

Lehtonen, A., Salonen, A. O., & Cantell, H. (2019). Climate change education: A new approach for a world of wicked problems. In J. W. Cook (Ed.), *Sustainability, Human Well-Being, and the Future of Education* (pp. 339–374). Palgrave Macmillan. https://doi.org/10.1007/978-3-319-78580-6_11

McLaren, L. (2013). *Educating for complexities: Using future focus education to approach the wicked problem of climate change*. [Master's thesis, Victoria University of Wellington]. http://researcharchive.vuw.ac.nz/handle/10063/3348

Moshirnia, A. (2007). The educational potential of modified video games. *Issues in Informing Science and Information Technology*, *4*, 511–521.

Noss R., & Hoyles C. (2017). Constructionism and Microworlds. In E. Duval, M. Sharples, & R. Sutherland (Eds.). *Technology Enhanced Learning* (pp. 29–35). Springer. https://doi.org/10.1007/978-3-319-02600-8_3

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings.* Kluwer.

Noss, R., Hoyles, C., Saunders, P., Clark-Wilson, A., Benton, L., & Kalas, I. (2020). Making constructionism work at scale: The story of ScratchMaths. In N. Holbert, M. Berland, & Y. B.

Kafai (Eds.), *Designing Constructionist Futures: The Art, Theory, and Practice of Learning Designs* (pp. 39–51). MIT Press. https://doi.org/10.7551/mitpress/12091.003.0007

Papert, S. (1972). Teaching children to be mathematicians versus teaching about mathematics. *Journal of Mathematics in Science and Technology, 31*, 249–262.

Riling, M. (2020). Recognizing mathematics students as creative: Mathematical creativity as community-based and possibility-expanding. *Journal of Humanistic Mathematics*, *10*(2), 6–39.

Rittel, H. W. J., & Webber, M. M. (1973). Dilemmas in a general theory of planning. *Policy Sciences*, *4*(2), 155–169. https://doi.org/10.1007/bf01405730

Ruthven K. (2014) Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In A. Clark-Wilson, O. Robutti, N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era. Mathematics Education in the Digital Era, vol 2* (pp. 373–393). Springer. https://doi.org/10.1007/978-94-007-4638-1_16

Salen K. 2007. Gaming literacies: A game design study in action. *Journal of Educational Multimedia and Hypermedia*, *16*(3), 301–322.

Sarama, J., & Clements, D. (2002). Design of microworlds in mathematics and science education. *Journal of Educational Computing Research*, *27*(1), 1–5.

Sinclair, N., & de Freitas, E. (2014). The haptic nature of gesture: Rethinking gesture with new multitouch digital technologies. *Gesture*, *14*(3), 351–374. https://doi.org/10.1075/gest.14.3.04sin

Vergnaud, G. (2009). The theory of conceptual fields. *Human Development, 52*, 83–94. https://doi.org/10.1159/000202727

Wilensky, U. (2020). Restructurations: Reformulating knowledge domains through new representational infrastructure. In N. Holbert, M. Berland, & Y. Kafai (Eds.), *Designing Constructionist Futures: The Art, Theory, and Practice of Learning Designs* (pp. 387–390). MIT Press.

Wilensky, U., & Papert, S. (2010). Restructurations: Reformulations of knowledge disciplines through new representational forms. In *Proceedings of the Constructionism 2010 Conference*.

Wing, J. M. 2006. Computational thinking. *Communications of the ACM*, *49*(3), 33–35. https://doi.org/10.1145/1118178.1118215